University of Crete – Department of Mathematics and Applied Mathematics

Problem Set No 6

Harmonic Analysis - Spring 2021-22 - M. Kolountzakis

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1. If $f \in L^1(\mathbb{T})$ and $N \in \mathbb{N}$ find the Fourier coefficients of the function $f(Nx)$ via those of $f(x)$.

2. If $f \in L^1(\mathbb{T})$ and $g \in L^\infty(\mathbb{T})$ show that

$$\lim_{n\to\infty} \frac{1}{2\pi} \int_0^{2\pi} f(t)g(nt)\,dt = \widehat{f}(0)\widehat{g}(0).$$

Show it first when f is a trigonometric polynomial. Then use the density of trigonometric polynomials in $L^1(\mathbb{T})$. Problem 1 will be useful.

3. If $a \in \mathbb{R} \setminus \{0\}$ and $0 < \sigma < 1$ show that the sequence $\{an^\sigma\}$ is uniformly distributed in $[0,1]$. ($\{x\}$ denotes the fractional part of $x \in \mathbb{R}$.)

Use Weyl's criterion. Approximate the sum $\sum_{n=1}^N e^{2\pi i k\{an^\sigma\}} = \sum_{n=1}^N e^{2\pi i k a n^\sigma}$ by the integral $\int_1^N e^{2\pi i k a x^\sigma}\,dx$ and bound their difference using the Mean Value Theorem in every interval of the form $[i, i+1]$.