

1. Prove that $e^x = 1 + x + x^2/2 + O(x^3)$ for $x \rightarrow 0$.

2. If $a_n = O(n)$ show that $\sum_{k=1}^n a_k = O(n^2)$.

3. If $f(x) = o(x)$ for $x \rightarrow 0$ show that

$$\frac{1}{1-f(x)} = 1 + o(x).$$

4. If $\epsilon > 0$ show that $\log x = o(x^\epsilon)$ for $x \rightarrow \infty$.

5. Show that $\sqrt{x + \sqrt{x}} = (1 + o(1))x^{1/4}$ for $x \rightarrow 0^+$.

6. Exactly one of the following relations is correct. Which one and why?

$$(a) \ 2^{o(n)} = o(2^n), \quad (b) \ 2^{O(n)} = O(2^n).$$