

1. Let  $q_1, q_2, \dots$  be an enumeration of the set  $\mathbb{Q} \cap [0, 1]$ . We define the set

$$E = \bigcup_{n=1}^{\infty} \left( q_n - \frac{1}{10^n}, q_n + \frac{1}{10^n} \right).$$

Show that the set  $[0, 1] \setminus E$  is not empty.

2. We define the set  $A \subseteq [0, 1]$  as follows. We define  $A_0 = [0, 1]$  and, for  $n = 1, 2, 3, \dots$ , we define the set  $A_n$  to be what is left from the set  $A_{n-1}$  if from each one of the intervals that make up  $A_{n-1}$  we remove the middle  $1/5$  of the interval. Having defined the sets  $A_n$  we finally define  $A = \bigcap_{n=0}^{\infty} A_n$ . Show that  $m(A) = 0$ .

3. We say that a set  $S \subseteq \mathbb{R}$  is of type  $G_\delta$  if it is the countable intersection of open sets, i.e. if there exist open sets  $G_n \subseteq \mathbb{R}$  such that  $S = \bigcap_n G_n$ . If  $E \subseteq \mathbb{R}$  show that there exists a  $G_\delta$  set  $S \supseteq E$  such that  $m(S \setminus E) = 0$ .