Problem Set No 3

---

1. If $f_n, f : [0,1] \to \mathbb{R}$ with $f_n \to f$ uniformly on $[0,1]$, show that $\int_{[0,1]} |f_n - f| \to 0$. Show that this is not true if the interval $[0,1]$ above is replace with $\mathbb{R}$.

2. Assume $0 \le f \in L^1(\mathbb{R})$. Show that $\int_{\{f>n\}} f \to 0$ for $n \to \infty$. Show also that for every $\epsilon > 0$ there exists $\delta > 0$ such that for every $E \subseteq \mathbb{R}$ with $m(E) < \delta$ we have

$$\int_E f \le \epsilon.$$

3. Suppose that $f : [-1,1] \to [0,+\infty]$ and that for every $t > 1$ we have that

$$m\{f > t\} \le \frac{1}{t^2}.$$

Show that $\int_{\mathbb{R}} f < \infty$.

We have (justify this)

$$\int f = \int_{\{f<1\}} f + \sum_{n=0}^{\infty} \int_{\{2^n \le f < 2^{n+1}\}} f.$$