1. Let $f(x) = \max\{0, x\}$ for $x \in \mathbb{R}$. Using the definition of Lebesgue integral for nonnegative functions show that $\int f = +\infty$.

2. If $A \subseteq \mathbb{R}$ and $f : A \to [0, +\infty]$ are such that

$$\int_A e^f < \infty$$

then show that

$$m\{f \geq \lambda\} \leq \frac{C}{e^\lambda}$$

for a constant C which does not depend on $\lambda > 0$.

3. If $f : \mathbb{R} \to [0, +\infty]$ and $E_1, E_2, \ldots \subseteq \mathbb{R}$, $E = \bigcup_{n=1}^\infty E_n$, then

(1) If the E_j are pairwise disjoint show that

$$\int_E f = \sum_{n=1}^\infty \int_{E_n} f.$$

(2) If $E_1 \subseteq E_2 \subseteq E_3 \subseteq \cdots$ show that

$$\int_E f = \sup \int_{E_n} f.$$

💡Use the Monotone Convergence Theorem.

4. If $f \in L^1(\mathbb{R})$ (not necessarily nonnegative) show that

$$\lim_{n\to\infty} \int_{[-n,n]} f = \int f.$$

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