University of Crete – Department of Mathematics and Applied Mathematics

Problem Set No 17

Turn in your solutions by 31/5/2020. See directions in the class webpage.

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1. If $f \in L^1(\mathbb{R})$ show that the Fourier Transform $\widehat{f}$ is uniformly continuous on $\mathbb{R}$.

2. If $f \in L^2(\mathbb{R})$ is the Riemann-Lebesgue Lemma valid?

3. Show that there exists a not-identically-zero C^∞ function which vanishes outside a bounded interval.

Use the function

$$\phi(x) = \begin{cases} e^{-1/x} & 0 < x \\ 0 & x \le 0 \end{cases}$$

4. If $1 \le p_1 \le p_2 \le \infty$ and $\frac{1}{p} = \frac{\theta}{p_1} + \frac{1-\theta}{p_2}$ show that for every $f : \mathbb{R} \to \mathbb{C}$

$$\|f\|_p \le \|f\|_{p_1}^{\theta} \|f\|_{p_2}^{1-\theta}.$$

Use Hölder's inequality as follows

$$\|f\|_p = \left\| |f|^\theta \cdot |f|^{1-\theta} \right\|_p \le \cdots$$

Harmonic Analysis - Spring 2019-20 - M. Kolountzakis