Turn in your solutions by 11/5/2020. See directions in the class webpage.

1. $C^1(\mathbb{T})$ is the space of functions on $\mathbb{T}$ that have a continuous derivative. Show that the quantity

$$\|f\|_{C^1} := |f(0)| + \|f'\|_\infty$$

is a norm on this space and that with this norm $C^1(\mathbb{T})$ is a Banach space.

Show also that the following quantity is also a norm (on the same function space)

$$\|f\|' := |f(0)| + \|f'\|_{L^2(\mathbb{T})},$$

but that the space is not complete with this norm.

Do we have convergence of the partial sums of the Fourier series on $C^1(\mathbb{T})$ (with the first norm)? Namely, is it true that for every $f \in C^1(\mathbb{T})$

$$\|S_N f - f\|_{C^1} \xrightarrow{N} 0?$$

The same question for the second norm.