All books and notes must be closed. No mobile phones should be on your person or even close to you. Leave your bags and phones by the teacher's desk.

University of Crete – Department of Mathematics and Applied Mathematics
Midterm exam - 21 November 2019
Duration: 2 hours

All curves are positively oriented unless otherwise noted.

---

1. (i) Find the subsets of $\mathbb{C}$ (possibly empty) determined by the relations:

$$(a)\quad \left|z^2\right| = |z|^3, \qquad (b)\quad |z| = |z-1| = |z+1|, \qquad (c)\quad \overline{z} - 1 = z + 1.$$

(ii) Find all z for which

$$\overline{z} = z^3.$$

Hint: watch the point 0.

2. (i) Show that the sum of the n-th roots of any complex number is 0.
(ii) Show that for all $z, w \in \mathbb{C}$ we have

$$|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2).$$

3. (ii) Using **only the definition of the derivative** show that the function z^2 is differentiable everywhere.
(ii) Show that the conjugate of the function in (i) is not differentiable except at 0. Here you are not restricted to the definition only.

4. Compute the integral

$$\oint_C \frac{dz}{z^2(z-1)^2},$$

where C is the circle $|z| = 2$.

5. Compute the integral

$$\oint_C \frac{e^{\pi z}dz}{(z^2+1)^2}$$

where C is the circle $|z| = 2$.

6. Show the inequality

$$\left|\oint_C \frac{e^z\,dz}{z^2+2}\right| \le e\pi,$$

where C is the right semi-circle of the unit circle.