1. Find the Taylor series of the function $\cos z$ around $\pi/2$.

2. Assume that $f = g'$ in the region $|z - z_0| < r$. Assume that f, g are analytic in this region. Assume also that $g(z) = \sum_{n=0}^{\infty} g_n (z - z_0)^n$ in that region. Find the Taylor series of f in terms of the g_n.

3. Find the Taylor series of the function $\operatorname{Log}(1 - z)$ around 0. Where does it converge?

4. Find the Taylor series of the function

$$f(z) = \frac{1}{1-z}$$

with center at 1/2. Determine in which disk this power series converges.

Repeat the same question but with center at 1/3.

Hint: Write $w = z - \frac{1}{2}$. Then

$$\frac{1}{1-z} = \frac{2}{1-2w} = 2\sum_{n=0}^{\infty} (2w)^n.$$

Repeat the same question but with center at 2.

5. Assume that f is analytic in the region $|z| < 1$ and that $f \equiv 0$ on the real axis. Show that $f \equiv 0$ in the region $|z| < 1$.

Hint: The values of f on the real axis are enough to determine all its derivatives at 0.

6. Find all the Laurent series (in all annuli defined by the singularities of the functions) around the point z_0 given.

(a) $f(z) = \frac{1}{1-z}, z_0 = 0,$ $\quad (b)$ $f(z) = \frac{1}{1-z}, z_0 = 2,$ $\quad (c)$ $f(z) = e^{1/z}, z_0 = 0,$ $\quad (d)$ $f(z) = \frac{z}{(z-1)(z-3)}, z_0 = 1.$

7. Suppose you have the Laurent expansion

$$f(z) = \sum_{n=-\infty}^{\infty} a_n z^n, \quad (r < |z| < R).$$

Find the Laurent expansion of the function

$$g(z) = (az^2 + bz + c) f(z).$$