1. Let C be the upper semicircle of the unit circle $|z| = 1$. Find the integral

$$\oint_C e^z \, dz.$$

Also the integral

$$\oint_C \overline{z^2} \, dz.$$

2. Prove that

$$\left| \oint_C \frac{dz}{z^2 - 1} \right| \le \frac{\pi}{3},$$

where C is the arc from 2 to $2i$ of the circle $|z| = 2$. Write the corresponding inequality if C is the arc of the same circle from 2 to $2e^{i\pi/4}$.

3. Let f be a polynomial of degree n:

$$f(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0, \quad a_0, a_1, \ldots, a_n \in \mathbb{C}.$$

Show that there is a constant $C > 0$ such that if $R \ge 1$ we have

$$\left| \oint_{C_R} f(z) \, dz \right| \le C R^{n+1},$$

where C_R is the semi-circle $|z| = R$, $\operatorname{Im} z > 0$.