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1. Prove the inequality

$$\left|e^{2z+i} + e^{iz^2}\right| \le e^{2x} + e^{-2xy}, \quad (z = x + iy \in \mathbb{C}).$$

2. Prove the inequality

$$\left|e^{z^2}\right| \le e^{|z|^2}, \quad (z \in \mathbb{C}).$$

3. We define the functions for $z \in \mathbb{C}$:

$$\cos z = \frac{1}{2}(e^{iz} + e^{-iz}), \quad \sin z = \frac{1}{2i}(e^{iz} - e^{-iz}),$$

$$\cosh z = \frac{1}{2}(e^{z} + e^{-z}), \quad \sinh z = \frac{1}{2}(e^{z} - e^{-z}).$$

Prove

$$|\sin z|^2 = \sin^2 x + \sinh^2 y, \quad |\cos z|^2 = \cos^2 x + \sinh^2 y, \quad (z = x + iy \in \mathbb{C}).$$

Then prove

$$|\sinh y| \le |\sin z| \le \cosh y, \quad |\sinh y| \le |\cos z| \le |\cosh y|, \quad (z = x + iy \in \mathbb{C}).$$

4. Prove that the function $e^{\overline{z}}$ is nowhere analytic.