

1. Find one value of  $\arg z$ .

$$z = \frac{-2}{1 + \sqrt{3}i}, \quad z = \frac{i}{-2 - 2i}, \quad z = (\sqrt{3} - i)^6.$$

2. Using the polar form show that

$$(i - 1)^7 = -8(1 + i).$$

3. Find the following roots in polar coordinates and show them geometrically.

$$(2i)^{1/2}, \quad (-1)^{1/3}, \quad (-16)^{1/4}, \quad 8^{1/6}.$$

4. Use de Moivre's formula to compute  $\cos 4\theta$  as a function of  $\cos \theta, \sin \theta$ .

5. If  $z \in \mathbb{C} \setminus \{1\}$  and  $n \in \mathbb{N}$  show that  $1 + z + z^2 + \dots + z^n = \frac{1 - z^{n+1}}{1 - z}$ . Then use this to show the formula

$$1 + \cos \theta + \cos 2\theta + \dots + \cos n\theta = \frac{1}{2} + \frac{\sin \frac{2n+1}{2}\theta}{2 \sin(\theta/2)}, \quad (0 < \theta < 2\pi).$$

6. Which curve in the complex plane is described by each of the following parametrizations?

$$z(t) = 1 + i + \sqrt{2}e^{it}, \quad 0 \leq t < 2\pi,$$

$$w(t) = 1 + i + (i - 1)t, \quad t \in \mathbb{R},$$

$$u(t) = it - (1 - t), \quad 0 \leq t \leq 1,$$

$$s(t) = t + t^2i, \quad t \in \mathbb{R},$$

$$S(t) = it - t^2, \quad t \in \mathbb{R}.$$

7. Find the limits, if they exist, of the following sequences

$$z_n = \frac{i}{n}, \quad w_n = (-i)^n, \quad u_n = \operatorname{Arg} \left( -1 + \frac{i}{n} \right), \quad v_n = e^{2\pi i n/5}, \quad s_n = \left( \frac{1 - i}{4} \right)^n.$$

8. Prove the continuity of the function  $f(z) = \bar{z}$  at every  $z_0 \in \mathbb{C}$ .

9. Prove the continuity of the function  $f(z) = z^2$  at every  $z_0 \in \mathbb{C}$ .

10. Prove the continuity of the function  $f(z) = \frac{1}{z^2}$  at every  $z_0 \in \mathbb{C} \setminus \{0\}$ .

11. Write the following functions  $\mathbb{C} \rightarrow \mathbb{C}$  in the form  $u(x, y) + iv(x, y)$ , where  $u, v \in \mathbb{R}, z = x + iy$ :

$$f(z) = \frac{z + i}{z^2 + 1}, \quad g(z) = \frac{2z^2 + 3}{|z - 1|}.$$

12. Prove that the following functions  $\mathbb{C} \rightarrow \mathbb{C}$  are nowhere differentiable

$$z \rightarrow \operatorname{Re} z, \quad z \rightarrow \operatorname{Im} z, \quad z \rightarrow |z|.$$