1. Find a few terms of the Laurent series of the function

$$\frac{e^{1/z}}{z^2-1},$$

around 0, and in all different annuli where this is defined.

2. Find the Laurent series of the function

$$\frac{z+1}{z(z-4)^3}$$

in the annulus $0 < |z-4| < 4$.

3. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty}(n+1)z^n$$

and to which function it converges.

4. Find the Taylor series of $\cos z$ around 0 differentiating termwise the Taylor series of $\sin z$.

5. Find the first 3 terms (3 smallest powers) of the Laurent series of

$$\frac{1}{e^z-1}$$

around 0. What is the annulus of convergence?

6. The function f is analytic in the domain $r < |z-z_0| < R$ and bounded there by M. If a_j, $j \in \mathbb{Z}$, are the coefficients of the Laurent series of f in that annulus show the inqualities

$$|a_j| \le \frac{M}{R^j}, \quad |a_{-j}| \le Mr^j, \quad (j = 0, 1, 2, \ldots).$$

7. Find the integral

$$\oint_C \frac{3z^3+2}{(z-1)(z^2+9)}\,dz$$

where C is the circle $|z| = 4$. Same for the circle $|z-2| = 2$.

8. If C is any circle which does not pass through the points $0, \pm 1$ how many different values can assume the integral

$$\oint_C \frac{1}{z+1} + \frac{10}{z} + \frac{100}{z-1}\,dz?$$

9. (a) If the function $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$ is continuous and even and C is the circle $|z| = 1$ show that

$$\oint_C f(z)\,dz = 0.$$

(b) If, in addition, the function is analytic in $\mathbb{C} \setminus \{0\}$ show the same if C is any simple closed curve going once around 0.
(c) Find a contiuous even function and a curve C as in (b) such that

$$\oint_C f(z)\,dz \neq 0.$$

10. Assume f is analytic at z_0 and has a zero there of order m. Show that the function $g(z) = f'(z)/f(z)$ has a simple pole at z_0 with residue m.