

1. Prove

$$|\operatorname{Re} z| + |\operatorname{Im} z| \leq \sqrt{2}|z|.$$

2. If $|z| = 2$ what is the maximum and minimum value of $|z^4 + 1|$?

3. Find all $z \in \mathbb{C}$ such that

$$|z + \bar{z}| = |z - \bar{z}|.$$

4. Find all $z \in \mathbb{C}$ such that

$$(1 + 2i)z + (3 - i)\bar{z} = 3 + 2i.$$

5. If $z, w \in \mathbb{C}$, $|z| \leq 1$, $|w| \leq 1$, show that

$$|z + w| \leq |1 + \bar{z}w|.$$

When do we have equality?

6. For which $z \in \mathbb{C}$ do we have

$$\operatorname{Re} z^2 = 3?$$

7. For which $z \in \mathbb{C}$ do we have

$$\operatorname{Im} z^2 = 3?$$

8. For which $z \in \mathbb{C}$ do we have

$$2|z| = z + \bar{z} + 4?$$

9. For which $z \in \mathbb{C}$ do we have

$$\operatorname{Re}(z + iz) < 2?$$