

1. Compute the integrals

$$(a) \int_1^2 \left(\frac{1}{t} - i\right)^2 dt, (b) \int_0^{\pi/6} e^{2it} dt, (c) \int_0^{\infty} e^{-zt} dz \ (\operatorname{Re} z > 0).$$

2. Compute the integral $\forall m, n \in \mathbb{Z}$:

$$\frac{1}{2\pi} \int_0^{2\pi} e^{imt} \overline{e^{int}} dt.$$

3. Find a differentiable function $f : [0, 1] \rightarrow \mathbb{C}$ such that for no $\xi \in [0, 1]$ it is true that

$$f'(\xi) = f(1) - f(0).$$

In this way you have shown that the mean value theorem fails for complex valued functions.

4. If Γ is the upper half of the unit circle $\{|z| = 1\}$ (oriented from left to right) compute the contour integral

$$\oint_{\Gamma} 1 + z^2 dz.$$

5. If Γ is the line segment from 1 to i compute the contour integral

$$\oint_{\Gamma} e^z dz.$$

6. If Γ is the line segment from 1 to 0 followed by the line segment from 0 to i find a parametrization of the curve Γ . Then compute the contour integral

$$\oint_{\Gamma} z^3 dz.$$