## University of Crete - Department of Mathematics and Applied Mathematics Problems No $21\,$

All curves are positively oriented unless otherwise noted.

1. Compute the integral

$$\int_{-\infty}^{\infty} \frac{x^3 \sin(ax)}{x^4 + 4} dx, \quad (a > 0).$$

**2.** Compute the integrals

(a) 
$$\int_{0}^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$$
, (b)  $\int_{0}^{\pi} \sin^{2n}\theta \, d\theta$ ,  $(n \ge 1)$ .

**3.** Using summation by parts, as we did in class, show Abel's inequality: If  $m \le \sum_{j=1}^k a_j \le M$ , for  $k = 1, 2, \dots, n$ , and  $b_1 \ge b_2 \ge \dots \ge b_n > 0$  then

$$b_1 m \le \sum_{j=1}^n a_j b_j \le b_1 M.$$