All books and notes must be closed. No mobile phones should be on your person or even close to you. Leave your bags and phones by the teacher's desk.

University of Crete – Department of Mathematics and Applied Mathematics
Midterm exam, 5 Nov. 2018 – Duration 2 hours.

All problems are equivalent in terms of grade.

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**1.** (i) Compute the cube roots of $-3+3i$. It is enough to give them in polar form.
(ii) Solve the equation $z^2-4z+5=0$.

**2.** (i) Prove the identity

$$|1+z\overline{w}|^2+|z-w|^2=(1+|z|^2)(1+|w|^2).$$

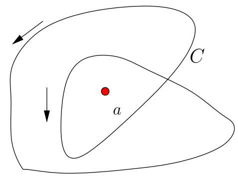
(ii) Compute the limit (here $t\in\mathbb{R}$)

$$\lim_{t\to+\infty}\frac{\cos(it)}{e^t}.$$

**3.**
Compute the integral (explaining your steps)

$$\frac{1}{2\pi i}\oint_C\frac{(z+a)dz}{z-a}$$

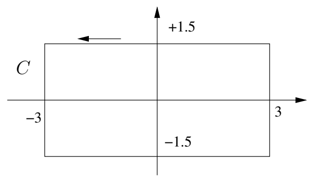
for the curve shown in the figure.




**4.**
Compute the integral (explaining your steps)

$$\frac{1}{2\pi i}\oint_C\frac{e^{z^2}dz}{z^2(z-4)(z-i)}$$

for the curve (rectangle) shown in the figure.




**5.** Find all $z\in\mathbb{C}$ for which the following series converges: $\sum_{n=0}^{\infty}\frac{1}{1+n^3}(z-2)^n$.

**6.** The function $f$ is analytic in $\mathbb{C}$ except at 1. It is also bounded in the set $\{z\in\mathbb{C}:\ |z|\le 10\}$. Prove that

$$\oint_{|z|=5}f(z)\,dz=0.$$

**7.** Find all pairs of complex numbers $z,w$ such that

$$|z|=|w|=1,\quad 1+z+w=0.$$

**8.**
Prove the inequality

$$\left|\oint_\Gamma\frac{dz}{z}\right|\le\pi.$$

The curve $\Gamma$ is the **open** polygonal line connecting the points
$R, R+iR, -R+iR, -R$.