Turn in your solutions by 27/4/2020. See directions in the class webpage.

1. If $f \in C^1(\mathbb{T})$ show that $\sum_{n\in\mathbb{Z}} \left|\widehat{f}(n)\right| < \infty$ (and thus that the Fourier series of f converges uniformly to f).

$\sum_{n\neq 0} \left|\widehat{f}(n)\right| = \sum_{n\neq 0} \frac{1}{|n|}\left|in\widehat{f}(n)\right|$.

2. Compute, as a function of $\alpha \in \mathbb{R}\setminus\mathbb{Z}$, a formula for the series

$$\sum_{n=-\infty}^{\infty} \frac{1}{(n+\alpha)^2}.$$

Let $f(x) = \frac{\pi}{\sin(\pi\alpha)} e^{i(\pi-x)\alpha}$. Show that $\widehat{f}(n) = \frac{1}{n+\alpha}$ ($n \in \mathbb{Z}$) and use Parseval's formula.

3. If $f(x) = x$, for $x \in [0, 2\pi]$, compute the Fourier coefficients of f and use Parseval's formula to compute the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$.