All books and notes must be closed. No mobile phones should be on your person or even close to you. Leave your bags and phones by the teacher's desk.

University of Crete – Department of Mathematics and Applied Mathematics
Midterm exam - 4 November 2019

All curves are positively oriented unless otherwise noted.
All problems are equivalent in grade and 8 problems give you the top grade.

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1. Describe and draw the subsets of $\mathbb{C}$ determined by the relations:

$$(a)\quad |z-i| = |z-1|, \qquad (b)\quad |z-i| = |z-1| = |z+1|.$$

2. Find a mapping $w = f(z)$ which maps the first region to the second in each of the following cases

$$(a)\quad \{z : \operatorname{Im} z \ge 2\} \to \{w : \operatorname{Re} w \ge 3\}, \qquad (b)\quad \{z : |z-1| \le 1\} \to \{w : |w-i| \le 2\},$$

3. Show that the sum of the cubic roots of any complex number is 0.

4. Without using the Cauchy-Riemann equations show that the function $f(z) = \operatorname{Re} z$ is nowhere differentiable. Use the definition of the derivative only.

5. Show the inequality

$$\left| e^{2z+i} - e^{iz^2} \right| \le e^{-2xy} + e^{2x}, \quad \forall z = x+iy \in \mathbb{C}.$$

6. Compute the integral

$$\oint_C \frac{dz}{z(z-1)^2},$$

where C is the circle $|z| = 2$.

7. Compute the integral

$$\oint_C \frac{dz}{z^2+1}$$

(a) where C is the circle $|z-i| = 1$, and (b) where C is the circle $|z| = 2$.

8. Show the inequality

$$\left| \oint_C \frac{e^{\overline{z}}\, dz}{z^2+1} \right| \le \frac{4\pi e^2}{3},$$

where C is the circle $|z| = 2$.

WARNING: The function in the integral is not analytic.

9. (Bonus points) In Problem 8 give a smaller upper bound.