1. Prove the inequality
$$\left| e^{2z+i} + e^{iz^2} \right| \leq e^{2x} + e^{-2xy}, \quad (z = x + iy \in \mathbb{C}).$$

2. Prove the inequality
$$\left| e^{z^2} \right| \leq e^{|z|^2}, \quad (z \in \mathbb{C}).$$

3. Prove
$$\left| \sin z \right|^2 = \sin^2 x + \sinh^2 y, \quad \left| \cos z \right|^2 = \cos^2 x + \sinh^2 y, \quad (z = x + iy \in \mathbb{C}).$$

Then prove
$$\left| \sinh y \right| \leq \left| \sin z \right| \leq \cosh y, \quad \left| \sinh y \right| \leq \left| \cos z \right| \leq \left| \cosh y \right|, \quad (z = x + iy \in \mathbb{C}).$$

4. Prove that the function $e^{\bar{z}}$ is nowhere analytic.

Complex Analysis - Fall 2019-20 - M. Kolountzakis