All curves are positively oriented unless otherwise noted.

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1. If f is analytic in $\mathbb{C} \setminus \{0, 1\}$ and $\sum_{n=-\infty}^{\infty} a_n z^n$ is its Laurent series for $|z| > 1$ show that the series is not finite towards $-\infty$, i.e., there are arbitrarily large $n > 0$ such that $a_{-n} \neq 0$.

2. (a) If f is an entire function and $g(z) = \overline{f(\overline{z})}$ show, using only the definition of the derivative, that g is also entire.

(b) If two entire functions coincide on the real axis show that coincide everywhere.

(c) If an entire function f is real on the real axis show that $f(\overline{z}) = \overline{f(z)}$.

3. Find the singularities and the corresponding residues for the functions:

$$(a)\ \frac{e^3 z}{z-2}, \quad (b)\ \frac{z+1}{z^2-3z+2}, \quad (c)\ \frac{\cos z}{z^2}, \quad (d)\ \left(\frac{z-1}{z+1}\right)^3.$$

4. If f has an isolated singularity at z_0 show that $\operatorname{Res}(f'; z_0) = 0$.

5. If f has a pole of order m at z_0 show that

$$\operatorname{Res}\left(\frac{f'}{f}; z_0\right) = m.$$