

All curves are positively oriented unless otherwise noted.

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1. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} (n+1)z^n$$

and to which function it converges.

2. Find the Taylor series of  $\cos z$  around 0 differentiating termwise the Taylor series of  $\sin z$ .  
3. Find the first 3 terms (3 smallest powers) of the Laurent series of

$$\frac{1}{e^z - 1}$$

around 0. What is the annulus of convergence?

4. The function  $f$  is analytic in the domain  $r < |z - z_0| < R$  and bounded there by  $M$ . If  $a_j, j \in \mathbb{Z}$ , are the coefficients of the Laurent series of  $f$  in that annulus show the inequalities

$$|a_j| \leq \frac{M}{R^j}, \quad |a_{-j}| \leq Mr^j, \quad (j = 0, 1, 2, \dots).$$