All curves are positively oriented unless otherwise noted.

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1. Find the Taylor series of the function $\cos z$ around $\pi/2$.

2. Assume that $f = g'$ in the region $|z - z_0| < r$. Assume that f, g are analytic in this region. Assume also that $g(z) = \sum_{n=0}^{\infty} g_n (z - z_0)^n$ in that region. Find the Taylor series of f in terms of the g_n.

3. Find the Taylor series of the function $\mathrm{Log}\,(1 - z)$ around 0. Where does it converge?

4. Find the Taylor series of the function

$$f(z) = \frac{1}{1-z}$$

with center at 1/2. Determine in which disk this power series converges.

Repeat the same question but with center at 1/3.

Hint: Write $w = z - \frac{1}{2}$. Then

$$\frac{1}{1-z} = \frac{2}{1-2w} = 2\sum_{n=0}^{\infty} (2w)^n.$$

Repeat the same question but with center at 2.

5. Assume that f is analytic in the region $|z| < 1$ and that $f \equiv 0$ on the real axis. Show that $f \equiv 0$ in the region $|z| < 1$.

Hint: The values of f on the real axis are enough to determine all its derivatives at 0.