

1. Using the function  $e^{f(z)}$  show that the maximum principle holds also for the real part of an analytic function  $f$ , and, of course, for the imaginary part as well: none of these two functions can have a local maximum. Show that they cannot have a local minimum either.

2. Find the maximum and the minimum (and where these values are taken) for  $\operatorname{Re} f$  where  $f(z) = e^z$  in the region

$$[0, 1] \times [0, \pi].$$

3. If  $f$  is analytic at  $z$  and  $f(z) \neq 0$  show that there is  $r > 0$  such that the value  $|f(z)|$  is not the minimum value of  $|f(w)|$  for  $|w - z| \leq r$ . *Hint:* Consider  $g(z) = 1/f(z)$ .

4. Find the maximum and the minimum absolute value of the function  $f(z) = (z - 2i)^3$  for  $|z| \leq 1$ .