1. Prove

$$|\mathrm{Re}\, z| + |\mathrm{Im}\, z| \leq \sqrt{2}|z|.$$

2. If $|z| = 2$ what is the maximum and minimum value of $\left|z^4 + 1\right|$?

3. Find all $z \in \mathbb{C}$ such that

$$|z + \overline{z}| = |z - \overline{z}|.$$

4. Find all $z \in \mathbb{C}$ such that

$$(1 + 2i)z + (3 - i)\overline{z} = 3 + 2i.$$

5. If $z, w \in \mathbb{C}$, $|z| \leq 1$, $|w| \leq 1$, show that

$$|z + w| \leq |1 + \overline{z}w|.$$

When do we have equality?

6. For which $z \in \mathbb{C}$ do we have

$$\mathrm{Re}\, z^2 = 3?$$

7. For which $z \in \mathbb{C}$ do we have

$$\mathrm{Im}\, z^2 = 3?$$

8. For which $z \in \mathbb{C}$ do we have

$$2|z| = z + \overline{z} + 4?$$

9. For which $z \in \mathbb{C}$ do we have

$$\mathrm{Re}\,(z + iz) < 2?$$