Georgia Institute of Technology
School of Mathematics

Practice final exam for MATH 2401, Sections J1 and J2

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No books, notes or calculators of any kind are allowed. Maximum number of points is 40 (10 for each problem). Duration of test is 2 hours 50 min.

Justify all your answers

1. Find the point of maximal curvature of the curve $y = \ln x$, for $x > 0$.
2. Find the points on the sphere $x^2 + y^2 + z^2 = 1$ that are the closest to and farthest from $(2, 1, 2)$. You must use the method of Lagrange multipliers.
3. Using a double integral and polar coordinates find the volume of the solid bounded below by the plane $z = 0$ and above by the surface

$$x^2 + y^2 + z^6 = 5.$$

4. Evaluate the line integral

$$\oint_C y^2\, dx$$

where C is the rectangle with vertices $(0, 0), (a, 0), (a, b), (0, b)$ oriented in the positive direction.
5. Let $\vec{h}(x, y, z) = (2xz + \sin y, x \cos y, x^2)$. Find a scalar function $f(x, y, z)$ such that $\vec{h} = \vec{\nabla} f$ and use it to evaluate the line integral

$$\oint_C \vec{h} \cdot d\vec{r}$$

where C is the curve given by the parametrization $\vec{r}(t) = (\cos t, \sin t, t)$, $0 \le t \le 2\pi$.
6. A homogenous wire of mass M winds around the z-axis as

$$C: \quad \vec{r}(t) = (a \cos t, a \sin t, bt), \quad 0 \le t \le 2\pi.$$

Find the length of the wire, the center of mass and the moment of inertia around the z-axis, in terms of the quantities a, b, M.
7. Suppose that f and g have continuous first order partial derivatives in a simply connected open domain Ω. Show that if C is any smooth simple closed curve in Ω, then

$$\oint_C \left(f(\vec{r}) \vec{\nabla} g(\vec{r}) + g(\vec{r}) \vec{\nabla} f(\vec{r}) \right) \cdot d\vec{r} = 0.$$