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System Evolution, Complexity and Control Theory Challenges

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Abstract

Complex Systems emerge in many disciplines and domains and have many interpretations and problems associated with them. Two major families expressing new forms of complexity are the Structure Evolving Systems (SES) and System of Systems (SoS) which are linked to design and operations of modern challenging applications in engineering, transport, energy and management systems. Existing methods in Systems and Control deal predominantly with Fixed Systems, that have been designed in the past, and which also have a well defined interconnection topology. The paradigm of Structure Evolving Systems (SES), expresses a new form of system complexity where the components, interconnection topology, input-output schemes may not be fixed, but may be subject to design and thus demonstrate features of structural evolution. The family of "System of Systems" (SoS) has emerged in many new fields of applications and they are characterized by the main property that the sub-systems have a degree of autonomy and the interconnection topology is generalized to a general composition rule, referred to as "systems play" which can take different forms including mathematical games. The common theme in those two new paradigms, SES and SoS is the notion of system evolution which runs through this research status paper. The paper addresses system and control challenges which lead to new Mathematical Systems and Control Theory problems in these two new families of systems by defining a number of generic clusters of system structure evolution problems, by establishing links with existing areas of control theory and by identifying new open mathematical problems. Different aspects of model evolution during the overall design are identified and an effort to identify the process of transition from Composite Systems (CoS) to SoS is made. The problems posed have a general systems and control character, but their study requires the solution of a number of challenging new mathematical problems. We focus here to the case of Linear Systems.

1. Introduction

Complex Systems emerge in many disciplines and domains and have many interpretations and implications. Different communities view complexity from their domain specifics and frequently the dialogue between communities such as biologists, physicist, economists, sociologists, computer scientists and engineers becomes difficult, or impossible. Mathematical Systems and Control Theory have the potential to provide the unifying framework (language and concepts) and the required tools (analysis, synthesis) for studying such problems, as long as it develops to handle some of the new challenges by providing solutions to a number of challenging mathematical and control theory problems linked to the emerging system paradigms. The paper deals with the forms of complexity inherent in two new system paradigms, the Structure Evolving Systems (SES), and the System of Systems (SoS). The SES family of systems emerges in many applications and are characterized by a variability of the system structure, its components and possibly its environment, in a way that defines an evolution of the system structure and the associated properties. Integrated Design (ID) [1, 2, 3] is a challenging task in many application areas (aeronautics, process systems etc) and defines the focus of our study of the SES paradigm [4] by providing a number of mostly open structural systems and control problems. The notion of "System of Systems" (SoS) [5],[6],[7] has emerged in many fields of applications from air traffic control to constellations of satellites, integrated operations of industrial systems. Such systems have the characteristic the interaction of many independent, autonomous systems, frequently of large dimensions, which are brought together in order to satisfy a global goal. They represent a synthesis of systems which themselves have a degree of autonomy, but this composition is subject to a central task and related rules frequently defined as "system play" expressing the subjection of subsystems to a central task. This paper aims to be of a review, as well as research status nature. It aims to identify the open issues which have a clear systems and control character with challenging associated new mathematical problems that may stimulate new lines of research.

Existing methods in Systems and Control deal predominantly with "fixed systems", that is those where the components, interconnection rule, measurement-actuation schemes, systems environment and control structures are fixed. Furthermore, the rule of connection is expressed as an interconnection topology (graph based). The process of overall design of a system (process synthesis - global instrumentation - control) has a cascade nature [2],[3], it is characterized by "early" and "late" stages and expresses a notion of "shaping", evolution of the model and thus of the resulting system properties. "Early design" requires evaluation of many alternatives using simple models and methodology [8],[1] whereas "late design" uses models of greater complexity and accuracy and requires more detailed evaluation of performance. Similar nature problems arise in the re-engineering of existing systems/networks in their upgrading to meet new requirements and performance demands. This may involve physical addition (growth), or removal (death) of parts of the system and represents evolution of a given system shell along a number of possible paths by intervening on the subsystem components, process synthesis/topology of interconnections, and selection of systems of measurement and actuation variables. Key questions that arise relate to modelling such forms of evolution, and then express model evolution in terms of the structural features and properties of the respective models. The major challenge is managing complexity involving the direction of such an evolutionary process along "paths" with desirable properties. This requires a methodology that is based on results characterizing the potential for evolution of system properties, explain the link of structure to invariants and performance indicators, characterize model uncertainty within a given system structure, define the "good" or "bad" potential for design, and provide means for addressing "structure assignment" problems. Responding to such tasks requires a new and richer form of Control Theory that is empowered to deal with the management of structure evolution by providing answers to a number of crucial new mathematical problems.

SoS on the other hand are complex multi-systems which exhibit features well beyond the standard notion of system composition [7]. They represent a synthesis of systems which themselves have a degree of autonomy, but this composition is subject to a central task and related rules. The term has been linked to problems of complex nature, but so far it has been used in a very loose way, by different communities with no special effort to give it a precise definition and link it to the rigorous methodologies concepts and tools of the Mathematical System Theory. Establishing the links with the traditional approaches is essential, if we are to transfer and appropriately develop powerful and established analytical tools to a field that is still unstructured and where little progress has been made in developing a generic and unifying methodology. There is need to place the concept of SoS within the standard framework of Systems Theory that is suitable for some subsequent further formal development (mathematical formulation). Such systems emerge in different and diverse domains and their classification, is also crucial, since different domains may require alternative modeling tools. A major challenge is to explain the difference of SoS from that of Composite Systems (CoS), which enables the generalization of the standard notion of interconnection topology (linked to composite systems) to the new notion of "systems play" [7] which now involves integrated system, that is systems with intelligence which can act as autonomous agents and can participate in mathematical games. This may allow seeing in certain cases the "systems play" as a game.

The main challenges in the SES family are linked to problems of describing evolution in systems design. Design is a cascade and complex process that is characterized by three main types of evolution. The first is linked to the natural evolution of the system structure as this is shaped through the design stages of conceptualization, process synthesis and global instrumentation and it is referred to as cascade structural evolution [2],[3]. The second stems from the need to address design and decision problems at "early" and "late" stages of system design (as part of an iterative design cycle) using models with a variability in their complexity and it is referred to as design time evolution [4]. The third deals with the type of evolution linked to re-engineering of a given structure and will be referred to as structural growth-death evolution [8]. For the SoS family the challenges stem from the effort to describe formally the notion of systems play [7] as an evolution of the standard notion of interconnection topology that is dominant for composite systems; this is also linked to the classification of the SoS family into subfamilies with common features stemming from the application domain they originate. The paper examines these forms of system evolution from a systems and control theoretic viewpoint and defines a number of research challenges for the structural system methodologies that can deal with such issues. The proposed structural approach is based on formulating problems stemming from application domains, defining appropriate representations and specifying a number of partial problems, which when combined may provide the essentials for a systems and control theoretic framework that may lead to their solution. Amongst the issues addressed are:

- Modeling issues in Early Process Design, model structure evolution and its representation
- Composite systems properties as functions of the interconnection graph
- System and Control problems in Systems Instrumentation
- System Redesign by changing subsystems and/interconnection topology
- System Graph evolution and Life-Cycle Design ("Growth-Death" problems)

— Characterization of systems play in SoS

The above areas express new major challenges for Control Theory represented by the SES and SoS families and introduce new mathematical problems. This issues raised depart considerably from the traditional assumption in control that the system is fixed and the dominant features are: **(i)**The topology of interconnections is not fixed but may vary through the life-cycle of the system (Variability of Interconnection Topology). **(ii)** The interconnection topology may be generalized to families of general relations that include mathematical games amongst others (System of Systems nature) **(iii)** The overall system may evolve through the early-late stages of the design process (Evolution through the Design Process). **(iv)** There may be Variability and/or uncertainty on the system's environment during the life-cycle requiring flexibility in organization and control strategies (Life-cycle Complexity). **(v)** The system may be large scale, multi-component and this may impact on methodologies and computations (Large Scale - Multi-component Complexity). **(vi)** There may be variability in the organizational Structures of the information and decision making (control) in response to changes in goals and operational requirements (Organizational Complexity Variability). **(vii)** The system boundaries may not be well defined (Uncertainty in Systems Boundaries).

In the context of SES some of the issues emerging are related to structure assignment and have been considered in some particular form in Control theory in the study of problems such as: zero assignment by squaring down [10],[11],[12], [13], [14], the dynamic cover problem of geometric theory [15],[16] etc. Such results contribute to the shaping of this new framework, but no effort to link them in a unifying framework has been made so far. Here, we introduce a number of challenging new problems for structural system theory linked to system evolution. This provides a framework for studying structure evolution in a systematic way, by defining key partial problems, and link them to systems and control design. The study of such problems relies on the classical structural methodologies for linear systems [15], [20],[21],[22],[23],[24],[25] and relies on a variety of mathematical methods such as linear algebra and matrix theory, algebra (including exterior algebra), graph theory, algebraic geometry, approximation theory, numerical analysis, algebraic computations etc. New problems in the study of SoS are linked more to methods from computer science, organization theory, game theory, social network modeling, network theory, etc. For both SES and SoS mathematical systems and control theory provide the framework linking the particular problems.

The paper is organized as follows: Section (2) provides a brief introduction to the notion of the system, system properties and structural characteristics which underpin the subsequent developments for SES and SoS classes. Section (3) provides a summary of the integrated design theme that generates the four different forms evolutionary processes i.e. the design time evolution, the cascade design evolution, redesign evolution and system growth/reduction evolution. In Sections (4), (5), (6) and (7) we discuss the representation, modeling, and the structural properties of the system under the evolutionary mechanisms. In each of the four families of evolutionary processes we define new mathematical challenges associated with such problems linked to the characterization of "model embedding", the evaluation of properties and invariant structure and their resulting evolution as a result of considered changes. In section (8) we review the basics of SoS and present the fundamental challenges in the characterization, representation of systems play.

2. The concept of the System and the Evolutionary Processes in Systems Design

2.1. The Notion of a System

The notion of a systems goes back to the Lakonian, and Pythagorean Kallicratides, in his work “Περι οικων ευδαιμονιας” (On the Happiness of Family) [26], [27]. He defines what is a “system” and explains it in terms of three examples: a) the system of dance in the singing societies, b) the system of the crew in a ship, and c) the system of the family, where the people are next of kin (have between them kindred relationships). This definition appears in the Anthology of J. Stobaei (Stobaei, J., Economicos, 16, 485) [26]. Modern developments of the notion have been influenced predominantly by the standard engineering paradigm. Efforts to extend the notion to new paradigms such as those of the business processes, data systems, biological systems, and emerging complex systems paradigms has been undertaken in the last 40 years leading to conceptual and formal developments of the subject [28], [29], [30]. Our task here is to reconsider existing concepts and notions from the general systems area [28], [29] and generalize them appropriately to make them relevant for the new challenges and then use them to define the notion of “System of Systems” in a subsequent section. We will also define the fundamentals required for describing the notions of system evolution under different transformations. We follow a conceptual systems approach that may lead to formal notions as described in [30]. Our work relies on existing methodologies, but aims at redefining notions, concepts and introduce new ones reflecting the needs of the new paradigms.

Definition 2.1 A system is an interconnection, organization of objects which are embedded in a given environment.

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This definition is general and uses as fundamental elements the primitive notions of: *objects, interconnection topology*, and *environment* and it is suitable for the study of “soft”, and “hard” systems. The concept of a system refers to the level of reality (physical or man-made construction) and this is an essential observation, to distinguish it from the notion of system model, which referrers to the sphere of abstraction. An object is a general unit (abstract, or physical) defined in terms of its attributes and the possible relations between them. For a given object, we define its *environment* as the set of objects, signals, events, structures, which are considered topologically external to the object, and are linked to the object in terms of a structure, relations between their attributes. The existence of the objects *environment* implies crossings of some imaginary boundary and such crossings indicate the separation of the system from objects external to its environment. The set of objects in a system are related between themselves and to the system environment through relationships referred to as *interconnection topology.* This defines the internal linking between the objects of the system and it is the *internal interconnection structure.* The linking of the objects of the system to objects outside the system's environment will be called *external interconnection topology.* The internal and external interconnection topology structure may be fixed or evolving and their nature gives to the system a specific character and identity. The nature of the external interconnection topology is crucial in defining the embedding of the system in its environment and it is the central notion in characterizing the difference between composite systems and system of systems. If $\mathcal{F}$ denotes the interconnection topology, S_a the system aggregate (collection of objects) and by $*$ the action of $\mathcal{F}$ on S_a we may represent the resulting system ,or simply as:

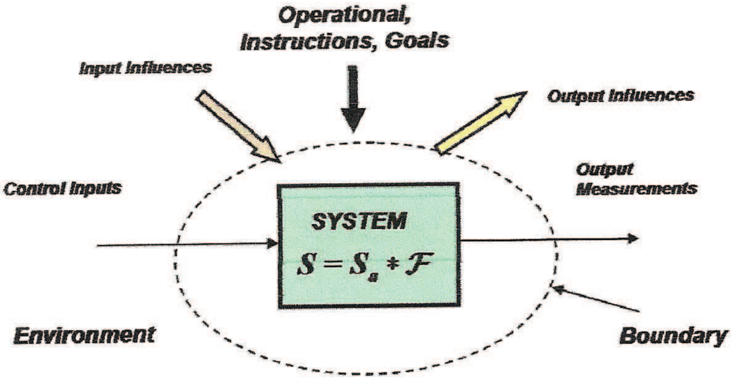

Figure 1: Description of an embedded system

Note that $\mathcal{F}$ usually denotes a directed graph and may be represented in the linear case by a structured matrix. An aggregate of systems leads to the creation of new forms of systems which may be either described within the framework of composite systems, or demonstrate additional features which add complexity to the description and may be referred to as system of systems.

2.2. **The Model Environment**

Throughout the paper we will consider for the sake of simplicity linear systems. We may have models where some of the internal variables are classified into potential inputs, outputs, internal variables and referred to as *oriented models* [4], or models where no classification has been made of the internal variables and are called *implicit models* [31]. All such models may be used for selection of effective sets of inputs and outputs, they are referred to as *progenitor models* [4] and they may be classified as:

— Internal Models (IM)

— External Models (EM)

— Internal-External Models (IEM)

(i) Internal Models [4]: These are described in terms of first order ordinary nonlinear equations and they are the standard state space descriptions of the implicit type $\underline{g}(\underline{\zeta},\dot{\underline{\zeta}}) = 0$, where $\underline{\zeta}$ is the vector of all internal model variables. In the linear case, the above reduces to matrix pencil model [32] defined by:

$$F\dot{\underline{\zeta}} = G\underline{\zeta} \tag{1}$$

When some inputs $\underline{\mathrm{u}}$, outputs $\underline{\mathrm{y}}$ have been defined, then we have correspondingly for the nonlinear is defined by $\underline{y} = \underline{h}(\underline{\xi},\underline{u})$, $\underline{q}(\underline{\xi},\dot{\underline{\xi}},\underline{u}) = 0$ and in the linear case by the singular model:

$$E\dot{\underline{\xi}} = A\underline{\xi} + B\underline{u},\ \underline{y} = C\underline{\xi} \tag{2}$$

When we have higher order derivatives, then auto-regressive descriptions are used [33].

(ii) External Models [33], [34]: If $\mathcal{V}, \mathcal{Z}$ denote the spaces of all potential inputs, measurements, referred to as extended input, output spaces respectively and $\underline{v}$, $\underline{z}$ are the corresponding p, q-dimensional vectors, then the external, or input-output map f is a function $f : \mathcal{V} \rightarrow \mathcal{Z}$ where $\underline{z} = f(\underline{u})$. For the case of linear, time invariant systems f is a convolution operator, which is represented in the Laplace domain by the $q \times p$ rational transfer function matrix $F(s)$, for which:

$$\underline{z}(s) = F(s)\underline{v}(s) \tag{3}$$

(iii) Internal-External Models : A large process is always synthesized by connecting sub-processes and the two fundamental ingredients of the composite system model are: **(a)** The topology (graph) of system interconnections $\mathcal{F}$, represented by a matrix F and **(b)** The family $\mathcal{M}$ of subsystem models which may be of any of the types discussed before. If S_a denotes the *aggregate* (direct sum) of the sub-processes and $\mathcal{F}$ the graph interconnection rule, then:

$$S_c = \mathcal{F} * S_a \tag{4}$$

which represents the composite system model and may be represented as a feedback type form [3], [4] as it will be explained in a following section. For the linear case S_a may be represented as a diagonal of transfer functions, and thus S_c becomes also a transfer function with a specific structure.

3. Systems Integration in Design and Operations as Evolutionary Processes

Systems integration [1], [2], [3] is a typical case of a multi-dimensional complex engineering problem that motivates many of the challenges of the SES and SoS families of systems. This problem has a multidisciplinary character and deals with the integration of [1], [2], [3]: (i) Engineering Design Stages, (ii) Process Operations, (iii) Engineering Design and Process Operations and (iv) Process Operations and Business Aspects. Each of the above areas has distinct dimensions relating to: (i) Physical Process, (ii) Signals and Operations and (iii) Data, IT, Software (iv) Operations and Business. The Physical Process Dimension deals with issues of design-redesign of the engineering system and predominantly relates to our driving paradigm the Integrated Design linked to problems of the SES family of systems. The ESPRIT II Project EPIC (1989) [8] has been an effort to provide a control based framework for the Early Process Design Stages of Continuous Chemical Processes and a description of the overall integration of design philosophy described as an evolutionary process was introduced in [3], [4] and it is elaborated here. The integration of the Operations, Signals, Data and Business aspects introduces new forms for complexity, many of them linked to the SoS family.

Integration of System Design

The general features of the technological stages of the overall system design are briefly considered first and are summarized by the diagram of Figure (1) [2]. The exchange of information illustrated in the above diagram between the different design stages has a short prediction horizon, as far as the impact on the subsequent design stages, and it is of a local character. This local character is dominated by the specialized skills, theory and techniques needed for a given engineering task. The ability to translate local decisions as actions assigning certain structure to the stage model is currently missing. The common engineering practice is dominated by simulations, trial and error and finally tested on a pilot plant. Accelerating the design process is crucial and this may be helped by developing a Global Coordination Theory for the design process. Our attention is focused on the purely technological stages of design, that is:

STAGE (I) : Process Synthesis
STAGE (II): Overall System Instrumentation (Global Instrumentation)
STAGE (III): Control Design.

The process synthesis - global instrumentation - control design stages have a cascade nature with feedback loops between the various sub-stages and have an iterative nature. The cascade nature of design is underlying the evolutionary process of model shaping, that drives the integrated design paradigm [2], [3], [4]. The cascade design process is dynamic in the sense that what it is feasible to achieve at a given stage is influenced by the decisions taken at the previous design stages. It is thus a characteristic feature of the cascade design process that decisions taken at one stage with local criteria, may, not necessarily be good as far as the overall design process. Understanding the evolution of the system, its behavior and properties through the successive design stages is a major challenge and handling it requires addressing the SES problems considered later on in the paper. The main design stages are [3], [4]:

Process Synthesis: This is an act of determining the optimal interconnection of processing units, as well as the optimal type and design of the units within a process system.

Global, System Instrumentation: This deals with the classification of system variables and the selection of the set and the distribution of inputs and outputs and its study revolves around the investigation of a number of fundamental system type problems. This is contrary to traditional instrumentation of a process that deals with the measurement, or implementation of action upon given physical variables.
The formation of structural characteristics of the overall process is reminiscent of an evolution process. The first stage, the process synthesis, acts as the parent gene and thus predetermines a possible range of key characteristics of the final process. Structural properties evolve, but not in a simple manner. Ideally, we would like to have them assigned in order to guarantee certain desirable characteristics and properties. This may not be possible and thus a more feasible design philosophy, is to direct the model evolution process towards final designs that may possess some desirable properties and avoid the formation of undesirable features that may penalize the final control design. The mechanisms of model structure formation in the early stages of design are not known yet. Developing some understanding requires addressing new challenging problems such as:

- **(a)** Design Time Evolution from “early” to “late” design stages
- **(b)** Numerical Dependent Evolution and model accuracy.
- **(c)** Cascade Design from Process Synthesis to Global Instrumentation.
- **(d)** Physical Growth, or Lifecycle Evolution.

The first notion of model evolution is linked to the general procedure in design, where we have a fixed interconnection structure but at the early stages we require *simple modeling* for sub-processes and physical interconnections; at the late stages of design *more detailed*, full dynamics models are required for sub-processes and physical interconnection structures. Here, we observe an evolution of the given structure of the system in the *design stage time* axis and this problem expresses the *Early-Late*

Design Variability of Model Complexity and corresponding accuracy [4]. The second type is linked to the level of numerical accuracy of the model that is used and it refers to the corresponding evolution of predicted properties for the respective model families. The third form of evolution is clearly connected to the cascade nature of the design process. The fourth form is linked to the physical growth, reshaping of the system during some re-engineering in response to different demands and it is a form that is linked to lifecycle issues. An additional form of complexity is introduced due to the integration of the system Operations, Data, Signals, Business Processes and their strong link to the physical system infrastructure; this leads to a new form of evolution referred to as system of systems (SoS).

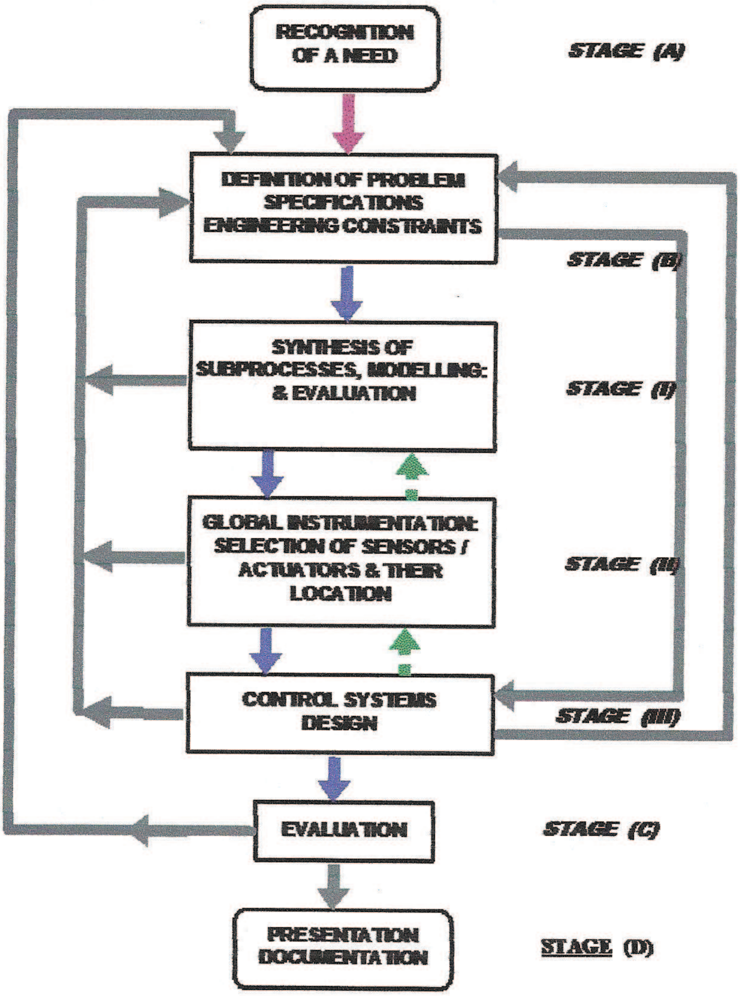

Figure 2: Simplified form of Engineering Design Process

4. Time Evolution in Systems Design and Control Theory

In many fields of engineering design and most predominantly in chemical processes [37], [1], [4], there is a need to visualise a complete design of the system, or many possible alternative designs at very early stages, evaluate them in some way and then select the most promising one. This requires the development of simple models, ability to create nests of variable complexity (early-late design) and methods to predict properties of the full system based on simple models. These issues lead to new problems in systems and control which require the study of new mathematical problem. A range of such open problems are considered here where we distinguish between cases where the graph structure is fixed and complexity of subsystem models varies and the case where there is a dimensional variability of the interconnection graph.

4.1. Design Time Evolution Nesting with fixed Interconnection Topology

A fundamental stage in the design process is the problem of Conceptual Modeling. This transforms Requirements and Objectives to sets of Preliminary Designs referred to here as *conceptual models* and denoted by $\mathcal{M}_i^c$ (see [37], [1]). The overall set of such models is denoted by:$\mathcal{M} = \{\mathcal{M}_i^c, i = 1, 2, ..., k\}$ where the basic elements in modeling are:

(i) The general interconnection rule defining the associated graph.

(ii) The early description of sub-processes in terms of simple models.

The exact nature of the graph depends on the stage of the design *(early, late)* and this is affected by the nature of models for local processes and the description of the physical interconnection streams. We may define the following notion of a graph associated with the system:

Definition 4.1 Let us denote every subsystem Σ_i by a pair of vertices $(e_i, w_i))$, denoting inputs and outputs, and an edge G_i providing an input-output description of Σ_i. If we denote by F_{ik} the physical (information) streams connecting the w_i output and the e_k input, defining an *edge*, the set $\{(e_i, w_i), G_i, F_{ik} : i, k = 1, 2, ..., \mu\}$ will be called the *kernel graph* of the system.

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This graph model is the simplest representation of the system structure, it is denoted as $\mathcal{M}^c$ and it is referred to as the *kernel model.* $\mathcal{M}^c$ contains the basic information linked to subsystems and physical streams, defines a primitive form of structure that stems from the conceptual model of the system and provides the minimal information on the physical interconnection topology. If $\mathcal{J}$ denotes the Graph of $\mathcal{M}^c$ and we denote by $\{\mathcal{M}_i^a, i = 1, 2, ...k\}$ the aggregate of the simple models of a given a-stage, we can denote by $\mathcal{M}_c^a$, the model defined as: $\mathcal{M}_c^a = J * diag\{\mathcal{M}_i^a, i = 1, 2, ..., k\}$. We consider two basic forms of system evolution. The first assumes variability of the dimensionality of vertices, whereas the transmittances have a fixed degree of complexity; this latter problem is referred to as vertex dimensionality evolution. The second assumes fixed dimensionality of vertices and variability in the description of complexity of the transmittance F_{ik} and it is referred to as *transmittance complexity evolution.* A more general case is that we have a combined *vertex dimensionality and transmittance complexity evolution.*

Vertex Dimensionality Evolution: At the initial stages the edges may be a scalar nature, that is (e_i, w_i) are scalars as well as F_{ik}; at later stages the dimensionality of physical interconnection streams may change, if more than one variable is associated with the physical streams, and we deal with (e_i, w_i) vectors and Matrix transmittances, as a result of increase in modelling requirements. This variability from 1-dimensional vertices, edges to many dimension vertices, edges respectively describes a form of evolution defined as *Dimensional Variability of Graphs.* Dimensional expansion and/or evolutionary expansion of the corresponding graph (vertices, edges expansion) imply that scalar nodes and edges in a graph may become vector nodes and vector edges and this represents a *Dimensional Graph Evolution* form. Instead of assuming a fixed $\mathcal{J}$ as above we may assume a set $\{\mathcal{J}\}$ and for every $\mathcal{J} \in \{\mathcal{J}\}$ we define $\mathcal{M}_c^a = J * diag\{\mathcal{M}_i^a, i = 1, 2, ..., k\}$ with $\{\mathcal{J}\}$ satisfying the graph evolution:

$$\mathcal{J}_0 \subseteq \mathcal{J}_1 \subseteq \mathcal{J}_2 \subseteq \mathcal{J}_3 \subseteq ...... \subseteq \mathcal{J}_{\mathcal{K}} \subseteq \mathcal{J}^* \tag{1}$$

The above nesting expresses the progressive enrichment of the initial graph that may be due either to increased local model complexity, and/or due to enhancement of description of the physical interconnection streams (dimensional expansion of graph branches). Such changes express distinct forms of evolution in the overall model and raise important new issues referred to as *graph evolution.* Fundamental issues related to the dimensional variability of the graph relate to the classification of the properties of the directed graph, which are independent/dependent on the dimensionality of the corresponding nodes. Such problems require proper formulation and lead to many open questions.

Dynamic Complexity Evolution: Starting from the kernel model, we may develop models of increasing complexity, generated from the same $\mathcal{M}^c$ model. This is done by preserving the generic structure of the interconnection rule, the *kernel graph*, fixing the dimensionality of vertices and successively using models with increasing complexity for the sub-process. This leads to the following nested set of models:

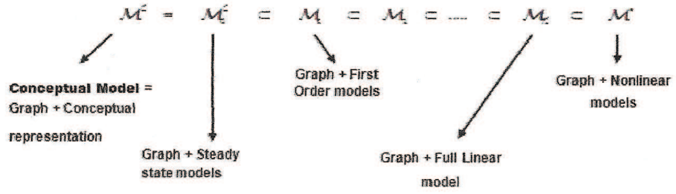

Figure 3: Nested set of models of variable complexity

Clearly, the process of model building continues beyond the construction of $\mathcal{M}^o$, which is the simplest nonlinear model. Note that the simplest nonlinear model $\mathcal{M}^o$ may be considered as a simple Voltera description, and subsequent nonlinear models with higher order Voltera descriptions [43] may be considered. This nesting expresses an evolution of the overall system model, parameterised by complexity (McMillan degree for linear systems) which is due to the evolution of *dynamic richness* of the subsystem models and it is due to the time dimension (*Early-Late*) of the design process referred to as *Design Time Evolution.* The resulting nesting of models is denoted in Figure (4).

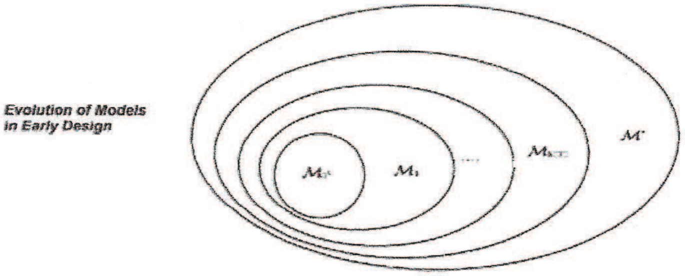

Figure 4: Model Embedding Process

Having generated such chains of models parametrised by their complexity issues of evolution of system structure and properties may be addressed. Issues that need addressing before we examine evolution relate to defining appropriate measures for complexity and how we can generate sequences of models. For the case of linear systems we will use the McMillan degree as the measure of complexity. The development of the sequences of models is more challenging. For real life systems the evolution from a simple model to an enhanced more detailed, and complex model requires in general detailed knowledge of the full system model and this is linked to the specific system. Developing sequences of models of variable complexity may be handled as follows:

(i) For systems described only by input output models we use model reduction techniques (see [38], [39], [41]) to generate the model chain.

(ii) For systems with an interconnection structure (kernel graph), and under the assumption of fixed dimensions for input, output structure of the subsystems, we use *structured model reduction* for simplification and development of the model chain, while we preserve the Graph Structure.

(iii) When there is combination of variable subsystem model complexity and variability of dimensionality of subsystem input-output structure (dimensional variability of graph), structured model reduction and *dimensional input-output simplification* (equivalent to input output squaring down [12], [14]) of fixed dynamics subsystem models are used to develop the model chain.

Note that there is a reversibility of the *model complexity evolution* and *model simplification approach.* Model Evolution and Model Reduction may become completely reverse processes, if we use fixed input, output subsystem structures and interconnection graphs. This may be referred to as *duality* between model reduction and model complexity evolution.

Challenges 4.1 Important research challenges are related to:

— *Representation of Variable Complexity Nesting Problems:* This involves describing the mechanisms of nesting by developing appropriate representations for graph evolution, measures for model complexity, and appropriate representation for overall model embedding.

— *Vertex Dimensionality Evolution:* This involves developing graph theory that can explain the transition from scalar to vector graph representations and then explain evolution of resulting system properties.

— *System Properties Evolution:* This involves studying the evolution of properties and structural characteristics in the chain of models and classify system properties according to: (i) their invariance, or dependence of the stage model and on early, or late appearance of property in the nesting; (ii) explain the evolution of system structure (algebraic and geometric invariants) in the model chain; (iii) define the required model complexity in an element of the chain for a given property to emerge.

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Challenging tasks are the representation of such forms of evolution, the simplification of graphs (in some sense), defining general measures of model complexity, and linking complexity, and genericity to system properties [44], [45]. Note that the generation of a model chain using model reduction techniques depends entirely on the technique that is used. The problem that arises is whether we can construct a *Universal Model Chain* that is independent from the specifics of the model reduction technique. This is considered next.

4.2. **Development of Universal Model Chains**

The description of a linear system in terms of the infinite Laurent expansion provides a natural way of deriving approximations of variable complexity by truncation of the infinite series. This natural way of introducing models of variable complexity is naturally linked to the classical problem of partial realization [22], [46]. It is assumed that the information available about a system S is an infinite sequence of Markov parameters $(\gamma_1, \gamma_2, ..., \gamma_N, ...)$. This input-output information is being used for the realization of a system (A, B, C) that would match only the first N terms of the infinite sequence. This realization is called partial realization. The partial realization establishes families of linear systems of variable dynamic complexity and this is why our attention is now focused on looking at this classical problem from a different perspective, that is the evolution in the family of models established by the partial realization.

State Space Realization: Let us consider a strictly proper transfer function $H(s) \in \mathbb{R}(s)^{m\times p}$. A linear system $\Sigma(A, B, C)$ with a state vector $\underline{x}(t) \in \mathbb{R}^n$, inputs $\underline{u}(t) \in \mathbb{R}^p$ and outputs $\underline{y}(t) \in \mathbb{R}^m$ described by:

$$\Sigma(A, B, C) : \dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t), \underline{y}(t) = C\underline{x}(t) \tag{2}$$

will be called a state space realization of $H(s)$ if $H(s) = C(sI - A)^{-1}B$ and n will be called its order. If $H(t) = C \cdot e^{At} \cdot B$ $(t \geq 0)$ is the impulse response matrix of $H(s)$ [48], then:

$$\left.\frac{d^{k-1}H(t)}{dt^{k-1}}\right|_{t=0} = \left.\frac{d^{k-1}(Ce^{At}B)}{dt^{k-1}}\right|_{t=0} = CA^{k-1}B = H_k = \gamma_k, k = 1, 2, ... \tag{3}$$

and the sequence $(\gamma_k)_{k=1}^{\infty} = (H_k)_{k=1}^{\infty} = (CA^{k-1}B)_{k=1}^{\infty}$ is the infinite sequence of *Markov parameters.* If $\gamma_1, \gamma_2, ...$ are the Markov parameters of $H(s)$, then the Laurent expansion of may be expressed as

$$H(s) = \sum_{i=1}^{\infty} \gamma_i s^{-i} \tag{4}$$

Remark 4.1 Note that the sequence of Markov parameters is invariant under state-space transformations.

■

Let us define the matrices Γ_c^k and Γ_o^k as in (5) and the infinite *Hankel matrix* as in

$$\Gamma_c^k = \left[\begin{array}{cccc} B & AB & ... & A^{k-1}B \end{array} \right], \Gamma_c^k = \left[\begin{array}{cccc} C & CA & ... & CA^{k-1} \end{array} \right]^T, k = 1,2,.. \quad (5)$$

$$H = \Gamma_o^\infty \Gamma_c^\infty = \left[\begin{array}{c} C \\ CA \\ \vdots \\ CA^K \\ \vdots \end{array} \right] \left[\begin{array}{ccccc} B & AB & \cdots & A^lB & \cdots \end{array} \right] =$$

$$= \left[\begin{array}{ccccc} CB & CAB & \cdots & CA^lB & \cdots \\ CAB & CA^2B & \cdots & CA^{l+1}B & \cdots \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ CA^kB & CA^{k+1}B & \cdots & CA^{k+l}B & \cdots \\ \vdots & \vdots & \cdots & \vdots & \ddots \end{array} \right] \quad (6)$$

Remark 4.2 ([48]) The system realization $\Sigma(A, B, C)$ is controllable if Γ_c^n has full rank and it is observable if Γ_o^n has also full rank. A realization which is both controllable and observable is said to be minimal and the dimension of the state-matrix (A) is equal to the McMillan degree of the transfer function $H(s)$.

■

We should note that the block elements in the Hankel matrix $H_{ij} = H_{i+j-1} = CA^{i+j-2}B$ are the Markov parameters. The McMillan degree of $H(s)$ (defined by the Smith-McMillan form [48]) determines the minimal number of states of a state-space realization which is also defined by:

Lemma 4.1 (49) Let $(\gamma_1, \gamma_2, ...$ be the Markov parameters of a strictly proper transfer function $H(s)$ defined as is (4.5). The minimal order of any state-space realization of $H(s)$ is given by $n_{\min} = rank(H)$ where H is the infinite Hankel matrix:

$$H = \left[\begin{array}{cccc} \gamma_1 & \gamma_2 & \gamma_3 & \cdots \\ \gamma_2 & \gamma_3 & \gamma_4 & \cdots \\ \gamma_3 & \vdots & \ddots & \ddots \\ \vdots & \vdots & \cdots & \ddots \end{array} \right] \quad (7)$$

■

Remark 4.3 For a strictly proper transfer function there will be a finite number of Markov parameters that result to a Hankel matrix of rank $n_{\min}$. The rest of the Markov parameters affect no change on the minimal order of the corresponding state-space realization of $H(s)$.

■

Formal power series and partial realizations

We consider for the sake of simplicity of the presentation the scalar case defined by a rational function $g(s)$. The Laurent expansion of this rational function generates an infinite series:

$$g(s) = \sum_{i=1}^{\infty} \gamma_i s^{-i} = \gamma_1 s^{-1} + \gamma_2 s^{-2} + \ldots \tag{8}$$

and thus defines an infinite sequence $\gamma = (\gamma_1, \gamma_2, ...)$ where the γ_i's being real numbers. Taking the first N ($N > 0$) terms of (8) we have a finite sequence $\{\gamma_N\} \equiv (\gamma_1, \gamma_2, ..., \gamma_N)$. A natural way to approximate the rational function $g(s)$ is to define a new infinite power series

$$\Gamma^{'}(s) = \sum_{i=1}^{\infty} \gamma_i s^{-i} = \gamma_1 s^{-1} + \gamma_2 s^{-2} + \ldots + \gamma_N s^{-N} + \gamma_{N+1} s^{-N+1} + ... \tag{9}$$

with the first N coefficients of the above power series being the corresponding γ_i's $\{\gamma\}_N \equiv (\gamma_1, \gamma_2, ..., \gamma_N)$ of the original sequence and the remaining $(\gamma^{'}_{N+1}, \gamma^{'}_{N+2}, ...)$ infinite number of terms being dependent on the finite sequence $(\gamma_1, \gamma_2, ..., \gamma_N)$ in some appropriate way that assumes that they do not provide any increase in the McMillan degree predicted by Lemma (4.1). Such an extension of the finite sequence is referred to as proper extension and the mechanisms of achieving this is based on the principle of not increasing the McMillan degree of the sequence that will be defined subsequently. This type of approximation is linked to the problem of *partial realization* [22] and provides a natural way to define models of variable complexity for rational functions.

In the following we shall refer to $\Gamma \equiv (\gamma_1, \gamma_2, ..., \gamma_N, \gamma_{N+1}, ...)$ as the *parent series*, the finite sequence $\{\gamma\}_N \equiv (\gamma_1, \gamma_2, ..., \gamma_N)$ as the *generator set* and the infinite sequence based on $\{\gamma\}_N$ which has been appropriately extended with $\gamma^{'}_{N+1}, \gamma^{'}_{N+2}, ...$ defined as linear functions $\Gamma^{'} \equiv (\gamma_1, \gamma_2, ..., \gamma^{'}{}_N, \gamma^{'}{}_{N+1}, ...)$ as a proper extension $\{\gamma_N\}$.

Remark 4.4 ([22],[46]) There always exist triplets of matrices $\Sigma \triangleq (A, B, C)$ of dimension $\mathbb{R}^{n\times n}$, $\mathbb{R}^{n\times 1}$ and $\mathbb{R}^{1\times n}$, called a realization of $\{\gamma\}_N$, respectively such that:

$$CA^{i-1}B = \gamma_i, i = 1, 2, ..., N, CA^{i-1}B = \gamma^{'}_{i}, i = N+1, ... \tag{10}$$

where n is the *dimension* of the realization. Every finite sequence $\{\gamma\}_N$ has a realization in the sense defined by (10) and the construction of such realizations is based on the rank properties of Hankel matrices. Of all possible realizations of a finite sequence $\{\gamma\}_N = (\gamma_1, \gamma_2, ..., \gamma_N)$ there is a family of realizations based on the $\Gamma^{'}$ proper extension with dimension $\delta(\gamma)$ minimal, defined by the rank properties of the sequence of Hankel matrices, and this is called the McMillan degree of $\{\gamma\}_N$.

■

Definition 4.2 ([22]) A *realization* $\Sigma_N \triangleq (A_N, B_N, C_N)$ of $\{\gamma_N\} = (\gamma_1, \gamma_2, \ldots, \gamma_N)$ based on the proper extension i.e., the infinite sequence $\Gamma^{'} \equiv (\gamma_1, \gamma_2, \ldots, \gamma^{'}_N, \gamma^{'}_{N+1}, \ldots)$ with dimension n and McMillan degree $\delta(\gamma)$ is called a *minimal partial realization* of the finite sequence $\{\gamma\}_N = (\gamma_1, \gamma_2, ..., \gamma_N)$.

■

Every realization $\Sigma_N = (A_N, B_N, C_N)$ of $\{\gamma\}_N = (\gamma_1, \gamma_2, ..., \gamma_N)$ (not necessarily a minimal) is related to a rational function $g_N(s)$, the *transfer function* of the finite sequence $\{\gamma\}_N$

$$g_N(s) = C_N(sI - A_N)^{-1}B_N \tag{11}$$

and the first N terms of the Laurent expansion of $g_N(s)$ will coincide with the corresponding N terms of $\{\gamma\}_N = (\gamma_1, \gamma_2, ..., \gamma_N)$ or the truncated power series. The process of considering $\{\gamma\}_N$ finite sequences of the infinite sequence Γ for varying values of N gives rise to a family of systems $\{\Sigma\}$ with corresponding transfer functions

$$\begin{aligned} \{\Sigma\} &= \left\{\Sigma_N : \Sigma_N \triangleq (A_N, B_N, C_N), N = 1, 2, ...\right\} \\ \{G\} &= \left\{g_N(s) \triangleq C_N(sI - A_N)^{-1}B_N, N = 1, 2, ...\right\} \end{aligned} \tag{12}$$

The construction of such systems linked to the finite sequences $\{\gamma\}_N$ has been developed in [22], [46] and there exist extensions of the results in the multivariable case. The development of a formal power series of a rational function $g(s)$ is linked to continued fraction based on the Euclidean Algorithm [51], and involve issues of recursiveness [40], [41]. The parameterisation of minimal partial realizations established in [51] in terms of input-output invariants [50] and the recursive description of the chain of models [40], [41] provides the natural means to characterize the properties of structural evolution as an evolution based on the properties of Kronecker invariants [21], [52].

Challenges 4.2 The study of the properties of $\{\Sigma\}$ and $\{G\}$ families is central to the effort to understand evolution of complexity in these chains. This requires:

— Parameterization of model complexity in the chain and possible extensions to nonlinear systems. Determine how the MacMillan degree changes in the model chain.

— Define the minimal order of complexity required for the emergence of different system properties, such as controllability, observability, stability etc. in any of the chains of models.

— Examine whether properties established for a certain value of complexity are preserved in models of higher value of complexity.

— Investigate the evolution of the invariant structure (Kronecker invariants etc) in the model chain.

■

Model Nesting, Small Numbers and Physical Simplification

Systems modeling involves assumptions based on the physics, and or the accuracy, order of numerical values of the model parameters. Such considerations introduce families of nested models which are ordered by some complexity measure based on the nature of the simplification. We may distinguish:

Small Numbers and Numerical Nesting: For linear state-space, or transfer function models, the issue that often arises, is how to handle numbers, which are very small and what is the impact of rounding off certain numbers of order less than a given order on the structural properties. It is essential to make a distinction between numbers which are small enough to be assumed equal to zero, and thus do not affect the overall structure of the system, or any of its properties, and numbers which are small, but represent a coupling, which has to be preserved. The small numbers, appearing in the A, B, C matrices, or in the set of Markov parameters $\left\{ \begin{array}{cccc} CB & CAB & CA^2B & \cdots \end{array} \right\}$ may be classified into structural which affect *structural* properties and *non-structural*, the removal of which has no effect on system properties. This classification introduces a numerical form of model nesting [53]. Removing small numbers is a form of Robust Structural Simplification, given that the structural properties of the original system have to be close to those of the reduced system.

Consider a state-space model (A, B, C) and let r be the element of maximal absolute value in (A, B, C). We can define the scaled model $(A^{'}, B^{'}, C^{'})$ as:

$$P^{'} = \frac{1}{r}\left[\begin{array}{cc} A & B \\ C & 0 \end{array} \right] = \left[\begin{array}{cc} A^{'} & B^{'} \\ C^{'} & 0 \end{array} \right] \tag{13}$$

For the elements of $P^{'}$ we have clearly $0 \leq |p_{ij}| \leq 1$. If $\varepsilon > 0$ is any small number, then all elements of $P^{'}$ for which $|p_{ij}| \leq \varepsilon$ defines a set, that will be called the *ε-power* of the original system. According to the value we select for ε we have a new model, the *ε-simplified model*, which is defined by the matrices $A_\varepsilon, B_\varepsilon, C_\varepsilon$, or $A_\varepsilon = r \cdot A_\varepsilon$, $B_\varepsilon = r \cdot B_\varepsilon$ and $C_\varepsilon = r \cdot C_\varepsilon$. The Boolean matrices associated with the ε-simplified model will define the structured model ε-structured model denoted by $\{P\}_\varepsilon$.

Robust Structural Simplification Problem: Different methods can be used to decide the significance of the value of ε and the final form of the ε-structured model $\{P\}_\varepsilon$. Amongst the possible methodologies we can use to analyse the effect of small numbers on the system properties are [55]: **(i)** Sensitivity Analysis Approaches; **(ii)** Robustness of Graph Structures; **(iii)** Degree of Controllability and Observability; **(iv)** System Based Metrics and Properties; **(v)** Matrix Perturbation Theory. All these methodologies mentioned adopt the same philosophy, which is to evaluate the effects of the different ε we choose on the structural properties of the system.

Consider a linear system represented by a state-space description $S(A, B, C)$, or by the set of *Markov Parameters* $\Sigma(H_1, H_2, ..., H_{n-1}, ...)$. We assume that the elements in the matrices involved are known only in terms of their relative order, but they are otherwise generic. This may be defined precisely as follows: Consider the set of positive real numbers $\{a_0, a_1, a_2, ...a_\mu,\}$ such that $a_0 > a_1 > a_2 > ... > a_\mu$ and define the following intervals:

$$E_0 = (\infty, a_0]\,, E_1 = (a_0, a_1]\,, E_2 = (a_1, a_2]\,, ..., E_\mu = (a_{\mu-1}, a_\mu]\,, E_{\mu+1} = (a_\mu, 0] \tag{14}$$

Definition 4.3 Let $M \in \mathbb{R}^{m\times n}$, and assume that the order of its elements (absolute values) are known only in terms of their membership of the sets $E_0, E_1, E_\mu, ..., E_{\mu+1}$ but they are otherwise generic. The matrix M will be called $\{a_0, a_1, a_2, \ldots, a_\mu\}$-*structured generic matrix* (i.e. $a_0 = 10^{-2}, a_1 = 10^{-3}, ..., a_\mu = 10^{-6}$) and the set of such matrices will be denoted by $\mathbb{R}^{m\times n}_{a_0,...,a_\mu}$.

■

Generation of Model Sequence: For a matrix $M \in \mathbb{R}^{m\times n}_{a_0,...,a_\mu}$ we may define the sequence of models:

M_0 : is obtained from M by setting all elements which are not in E_0 equal to zero.

M_1 : is obtained from M by setting all elements which are not in $E_0 \cup E_1$ equal to zero.

$$\vdots$$

M_μ : is obtained from M by setting all elements which are in $E_{\mu+1}$ equal to zero, or equivalently all elements not in $E_0 \cup E_1 \cup ... \cup E_\mu$ equal to zero.

■

The above process creates from M a set of matrices $M_0, M_1, M_2, ..., M_\mu$ which together with M define a nesting condition denoted by:

$$M_0 \subset M_1 \subset M_2 \subset ... \subset M_\mu \subset M_{\mu+1} \tag{15}$$

The relation "$\subset$" means that M_v is obtained from M_{v-1} those elements of M which have an absolute value in the interval $(a_{\nu-1}, a_\nu]$. This ordering on the M matrix will be called an $\{a_0, a_1, a_2, ..., a_\mu\}$ - *induced nesting* and this numerical nesting leads to families of State Space and Markov Parameter Nested Models which may be denoted by

$$\begin{aligned} \{S\}_\varepsilon &= S_j(A^j, B^j, C^j), j = 1, 2, ..., \mu \\ \{\Sigma\}_\varepsilon &= \Sigma_j(H_1^j, H_2^j, ...H_{n-1}^j, ...), j = 1, 2, ..., \mu \end{aligned} \tag{16}$$

The state space and the input-output modelling based on the Markov parameters provides different approaches for the study of robustness and sensitivity. The Markov parameters approach is naturally linked to the partial realization problem [22], [41] and this in turn implies some further evolution of structural properties based on the predicted McMillan degree of the partial realization [54], [55].

Challenges 4.3 *System Properties and Model Embedding:* For the families $\{S\}_\varepsilon$, $\{\Sigma\}_\varepsilon$ systems generated by any method of chain generation, a number of systems and control challenges emerge such as:

- Define the minimal value of the order ε, or physical modelling assumptions required for the emergence of different system properties, such as controllability, observability, stability etc. in any of the chains of models.
- Investigate how the McMillan degree varies as a function of ε, physical assumptions in families such as $\{\Sigma\}_\varepsilon$ in terms of ε.
- Examine whether properties established for a certain ε are preserved in models of higher accuracy ε.
- Investigate the existence of properties which are dependent, or independent on the selected value of ε, or the nature of other modelling assumptions.

■

Physical Simplification: Systems modelling based the physics of sub-processes and interconnection topology is based on assumptions relating to the relevance of different types of physical elements (assumptions whether for instance a resistance is significant or not, an element is linear or nonlinear etc). By ordering the significance of different physical elements and their respective values, nests of physical models may be obtained which preserve the natural topology of interconnections. Such nests are similar in nature to those depending on numerical value ordering. The fundamental difference is the preservation of the natural interconnection topology. The study of related properties of the chains may be based on modelling tools developed for network theory [56].

5. Process Synthesis: The evolution of the Aggregate to the Composite System

The fundamental problem in systems theory is the evolution from the *aggregate* to the *composite* system as the result of the action of the composition rule, referred as interconnection topology. In engineering this is referred to as process synthesis and it is usually addressed using methodologies linked to the particulars of the application area. The development of a generic synthesis framework based on a system theoretic methodology that transcends the different application areas is a challenge and has been addressed recently for the linear case in [3]. Energy considerations, and behaviours [57] have been used for the modelling of composite systems, whereas the traditional network synthesis together with the completion of the analogy between electrical and mechanical domains have been important new developments in this area [58]. In this section, we consider the linear case and develop the reduction of process synthesis to an equivalent feedback design problem [3] using the standard composite system description and its particular characteristics based on the nature of the physical interconnection streams and the selection of the local input and output structure [30]. This provides a representation of the synthesis as generalised feedback design problem, allows model simplification in systems with a clear interconnection structure and provides means to intervene in the shaping of system properties by design of the interconnection topology.

Definition 5.1 Given a set of sub-systems $\{\Sigma_i, i = 1, 2, \ldots, \mu\}$ represented by models $\{\mathcal{M}_i,\ i = 1, 2, \ldots, \mu\}$ of a certain type and with a given interconnection rule $\mathcal{F}$ (described by a graph). We define as:

(i) The *Aggregate System*: $\Sigma_a = \Sigma_1 \oplus \Sigma_2 \oplus ... \oplus \Sigma_\mu$ with $\mathcal{M}_a = block - diag\,\{\mathcal{M}_1, \mathcal{M}_2, ...\mathcal{M}_\mu\}$ as the corresponding aggregate model.

(ii) The *Composite System* is then defined by $\Sigma_c = F * \Sigma_a$, where $*$ denotes the action of F on Σ_a.

■

Sub-processes enter the composite structure, by the interconnection of local variables (subsystem connecting inputs, outputs and effective control inputs and measured outputs) and this affects drastically the overall properties of the composite system. The focus in this area has been on the study of properties of composite systems without seeing the interconnection scheme and the selection of local input, output structure as design parameters which affect the overall system structure and properties. Linking model composition to feedback was initiated in [59] and subsequently developed in [3]. The definition of the composite from the aggregate by the action of the interconnection topology raises important questions, which are linked to: **(i)** The representation of the composite system [3]; **(ii)** The relationships between the structure and properties of the aggregate and the composite in terms of the characteristics of the interconnection topology [60]. The development of the general scheme requires certain assumptions described below:

(a) Local Well Connectedness Assumption (LWCA): The physical linking of a subsystem Σ_k to the rest of the subsystems implies that there is a connecting input vector $\underline{e}_k$ having as coordinates all variables connected directly to at least one subsystem output, or external variable (manipulated, or disturbance) and having as connecting output vector $\underline{z}_k$ the vector with coordinates all variables which feed to at least one of the subsystems. We assume that the transfer functions $H_k(s) : \underline{e}_k \to \underline{z}_k$ are well defined and they are proper. These assumptions are referred to as *Local*

Well Connectedness (LWC) and $H_k(s)$ is the *k-th connecting transfer function.* The aggregate system Σ_a is represented by the transfer function matrix $H(s) = block-diag\{H_k(s), k = 1, ..., \mu\}$.

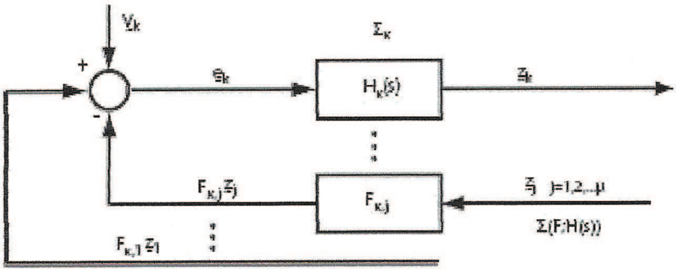

Figure 5: The global well formedness assumption

(b) Local Well Structured Assumption (LWSA): For every subsystem we shall denote by $\underline{u_k}$, $\underline{y_k}$ the effective input, output vectors. We shall assume that $\underline{y_k}$ is a sub-vector of $\underline{w_k}$ in the sense that $\underline{y}_k = K_k \underline{z}_k, K_k \in \Re^{p_k \times q_k}$, $p_k \leq q_k$ and that $\underline{v}_k$ is a reduced vector of $\underline{e}_k$ in the sense that:

$$\underline{e}_k = \underline{f}_k + L_k \underline{u}_k = \underline{f}_k + \underline{v}_k \in \mathbb{R}^{p_k},\ p_k \leq q_k \tag{1}$$

where f_k is some vector of dependent variables and the coordinates in u_k are independently assignable (control, or disturbance) variables. This assumption is referred to as *Local Well Structured Assumption.*

(c) Global Well Formedness Assumption (GWFA): Consider the aggregate system $\Sigma_\alpha = \{\Sigma_k, k = 1, 2, ..., \mu\}$ under the Local well formedness and local well structured assumptions [59]. The composite system is *globally well formed*, If the interconnection rule $F : \underline{e}_1 x ... x \underline{e}_\mu \to \underline{z}_1 x ... x \underline{z}_\mu$ represented above satisfy the assumptions:

(i) Its output is $\underline{z} = \left[\underline{z}_1^t, ..., \underline{z}_\mu^t\right]^t$ and if $\underline{v}_k$ are external vectors its inputs $\underline{e}_k$ are $\underline{e}_k = \sum_{j=1}^{\mu} F_{kj}\underline{z}_j + \underline{v}_k$, F_{kj} real.

(ii) The transfer function from $\underline{v} = \left[\underline{v}_1^t, ..., \underline{v}_\mu^t\right]^t \to \underline{e} = \left[\underline{e}_1^t, ..., \underline{e}_\mu^t\right]^t$ is defined. If

$$\begin{gathered}\underline{e} = \underline{v} + F\underline{z},\ \underline{z} = H(s)\underline{e},\ \underline{v} = L\underline{u},\ \underline{y} = K\underline{z} \\ F = [F_{kj}]_{k,j\in\mu}, K = bl.diag\{K_i, i \in \mu\}, L = bl.diag\{L_i, i \in \mu\}, \\ \underline{u} = [\underline{u}_1^t ... \underline{u}_\mu^t]^t, \underline{y} = [\underline{y}_1^t ... \underline{y}_\mu^t]^t\end{gathered} \tag{2}$$

and the composite configuration is represented as the feedback configuration of Fig. (6). Note that condition (c.ii) above implies that $I - FH(s)$ is an invertible matrix. It is clear that the interconnection graph acts as feedback and the selection of effective inputs, outputs is represented as input, output constant compensators and the composite transfer function is

$$G(s) = K\widehat{H}(s)L, \text{where} \widehat{H}(s) = H(s)\{I - FH(s)\}^{-1} \tag{3}$$

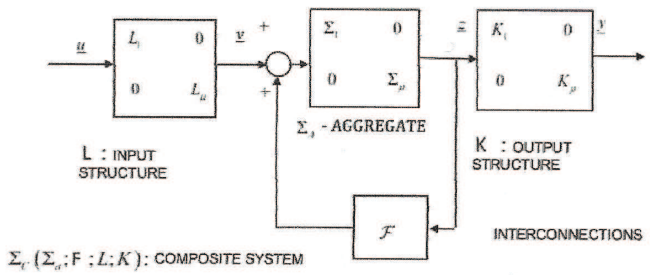

Figure 6: Equivalent Feedback Configuration

The above expresses the composite system as the composite action of decentralised input and output reduction (squaring down operation), represented by the input, output transformations K, L respectively and of an internal feedback F, representing the topology of the interconnections. The matrix $\widehat{H}(s)$ s referred to as *progenitor model* of the composite system. The actions of K, L are usually referred to as *Model Projection (MP)* operations and are forms of "squaring down" [12], [14]. The representation of the interconnecting topology as internal feedback provides the means for linking the properties of aggregate and the composite system and presents the synthesis problem as part the feedback theory. An important special case is described below [3]:

(d) Completeness Assumption: The well formed composite system of Fig.(6) will be said to be *complete*, if the following two further conditions hold true: **(i)** Every effective subsystem output $\underline{y}_k$ satisfies the condition $\underline{y}_k = Q_k \underline{z}_k$, Q_k square invertible. **(ii)** Every external subsystem vector $\underline{v}_k$ has as many independent coordinates as the dimension of e_k input vector, i.e. $e_k = R_k \underline{v}_k$ with R_k square and invertible.

Remark 5.1 As a result of completeness the composite and the aggregate are output feedback and input, output coordinate transformation equivalent and thus they have the same basic structural characteristics.

■

Guaranteeing the validity of the above assumptions, is both a matter of modelling and selection of input, output schemes; in fact, each assumption is linked to problems of: (i) design of individual sub-processes; (ii) identifying the effective connecting inputs, outputs $\underline{e}_k, \underline{z}_k$ and the potential control variables and outputs $\underline{u}_k, \underline{y}_k$; defining the connecting F_{kj} matrices such that $I - FH(s)$ is non-singular. When there is flexibility in the design of F, the objective may be extended in designing F such that the resulting model $\widehat{H}(s)$, referred to as *progenitor model*, is stable. An important problem with implications on the evolving structure is defined below:

Decentralised Model Projection Problems (DMPP): The general configuration of Fig. (6) suggests that the final selection of inputs, outputs is selection of L and K are block diagonal to assign structural characteristics as those defined by the Kronecker structure [52] and addressed on the progenitor model $\hat{H}(s)$.

Challenges 5.1 The system synthesis problems defined above introduce a number of systems a control problems such as:

— Explain system structure evolution as a function of F, L, K matrices.

— Extend the system composition framework and related properties to the case of nonlinear systems.

— Use the synthesis framework to develop nest of structured variable complexity models and develop methodology for structured model reduction.

■

The above problems are linked to process synthesis, but the way they are addressed have a clear control theory flavour and characterise a family of *Model Composition Problems (MCP)*. Such problems are linked to the interconnection scheme design (F matrix design) and are connected to the local input, output structure selection (L, K matrices design) and express model evolution with instruments the interconnection topology and the selection of inputs and outputs at the subsystem level.

6. Cascade Design Evolution and Control Theory

The selection of inputs and outputs is a crucial design task that shapes the system model and it is one of the central system evolution mechanisms. This study revolves around the study of four fundamental problems which are: **(i)** Model Orientation Problems (MOP); **(ii)** Model Projection Problems (MPP); **(iii)** Model Expansion Problems (MEP) and **(iv)** Local- Global Structure Problems (LGSP). These problems have a clear model shaping role, each one of them defines a form of system evolution expressed as shaping of the invariant system structure [52] and their study is reduced to problems of Control Theory and Design [3],[4]. These aim to develop design tools for assisting the "good" shaping of the system model structure in the global instrumentation (GI) stage of design. The distinguishing feature of GI as far as model shaping is that it acts on the shaping of the input-output structure, rather than the interconnection graph, shaped by process synthesis as described previously. The (iv) cluster of problems has already been addressed and expresses the interaction between GI and process synthesis. These key problems are considered below.

6.1. Model Orientation Problems

Physical modelling based on basic laws and use of interconnection topology may be used for large families of systems. If all important variables are included and there is no effort to guarantee their minimality, and their classification into inputs, internal variables is made, the emerging descriptions are referred to as implicit. In the case of first order differential descriptions they correspond to the matrix pencil, [32],[31],[63], or generalised autonomous description [32],[64]

$$S(F,G) : Fp\underline{\zeta} = G\underline{\zeta},\ F, G \in \mathbb{R}^{\tau\times\upsilon},\ \underline{\zeta} \in \mathbb{R}^{\upsilon} \tag{1}$$

or to the autoregressive description [62]

$$H(p)\underline{\xi} = 0,\ H(p) \in \mathbb{R}[s]^{\mu\times\nu},\ \underline{\xi} \in \mathbb{R}^{\nu} \tag{2}$$

where, p is the differentiation, or shift operator and $\underline{\zeta}, \underline{\xi}$, are vectors of all problem variables. The natural operator associated with such descriptions are the matrix pencil $sF - G$ [65], or the polynomial matrix $H(p)$ and thus their study relies on the structure of $sF-G$ or of general polynomial matrices. For control, as well as handling issues of creating composite structures, it is important to classify the variables in

$\underline{\zeta}$ or $\underline{\xi}$ into internal variables, or states $\underline{x}$, assignable, or control variables $\underline{u}$, and measurement, or dependent variables $\underline{y}$. The general problem of the classification of systems variables as inputs and outputs is referred to as model orientation problem (MOP). In many systems, the orientation is not known, or that depending on the use of the system the orientation changes. Questions such as, when is a set of variables implied, or not anticipated by another, or when is it free, have to be answered, if model orientation criteria based on the nature of the process are to be derived; the specific use of the system may provide additional model orientation criteria. It may happen, that the above two types of criteria do not provide a unique solution to model orientation; note that for each alternative orientation we have a different system model which expresses an evolution of the original implicit description. The free version of MOP for matrix pencil models (unconstrained by design specifications is defined as:

Definition 6.1 Given the model $S(F,G): Fp\underline{\zeta} = G\underline{\zeta}$, $F,G \in \mathbb{R}^{\tau\times\upsilon}$, $\underline{\zeta} \in \mathbb{R}^{\upsilon}$ define a transformation, $Q: \zeta = Q\tilde{\zeta}$, $Q \in \mathbb{R}^{\upsilon\times\upsilon}$, $|Q| \neq 0$, where $\zeta = \left[\underline{x}^t, \underline{u}^t, \underline{y}^t\right]$ where $\underline{x} \in \mathbb{R}^n$, $\underline{u} \in \mathbb{R}^p$, $\underline{y} \in \mathbb{R}^m$ such that $S(F,G)$ is equivalent to

$$\begin{bmatrix} p\hat{E} - \hat{A} & -\hat{B} \\ -\hat{C} & 0 \end{bmatrix} \begin{bmatrix} \underline{x}(t) \\ \underline{u}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -\underline{y}(t) \end{bmatrix} \tag{3}$$

■

The system $S(\hat{E},\hat{A},\hat{B},\hat{C})$ is called an orientation of $pF - G$, and it is in general of the singular type [31],[66] and $\Sigma(F,G)$ denotes the family of all such systems. Determining the conditions under which $S(F,G)$ may be reduced to singular, $S(\hat{E},\hat{A},\hat{B},\hat{C})$ or regular $S(A,B,C,D)$ descriptions is part of the study of model orientation and it is in a way a problem of partitioning of the Kronecker set of invariants of the matrix pencil $pF - G$ [52],[67].

Defining subsystems of $S(A,B,C,D)$ by reduction of the input, output structure such that the reduced system $S(A,B',C',D')$ has desirable properties is referred to as *input-output structure reduction problem* (I-ORP) which includes problems such as the squaring down [12] and are special forms of model projection problems considered in the following section. The family $\Sigma(F,G)$ contains more than one solution. Such solutions may be classified according to the invariant structural characteristics of the corresponding orientation, as well as the input, output type properties of the resulting oriented model. An important issue in selecting oriented models is the issue of model minimality [68],[69] which is equivalent to selecting a minimal number of internal variables. Issues of minimality, as well as assignment of desirable structural characteristics are important criteria which have to be used in the parameterisation of the $\Sigma(F,G)$ families.

A similar treatment may be given for the autoregressive form (2) and this leads to Rosenbrock type representation [25]. The external behaviour, or autoregressive description of (2) defines the set of all external-variable trajectories $\underline{\xi}(t)$ satisfying (2). A first effort to introduce orientation is expressed by the introduction of some internal variables, expressed by the partitioning of the vector $\underline{\xi}(t)$ as $\underline{\xi}(t) = \left[\underline{w}^t, \underline{u}^t, \underline{y}^t\right]^t$ and this leads to the AR/MA class described by the reduction of $H(p)$ to the Rosenbrock [25] system matrix representation

$$\begin{bmatrix} T(p) & -U(p) \\ -V(p) & -W(p) \end{bmatrix} \begin{bmatrix} \underline{w}(t) \\ \underline{u}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -\underline{y}(t) \end{bmatrix} \tag{4}$$

where all matrices are polynomial, with $T(p)$ square and invertible. In the Laplace domain the corresponding transfer function matrix $G(s)$ is represented as $V(s)T(s)^{-1}U(s) + W(s)$. The orientation problem may now be addressed in a more general setup where first we model the system behaviour, in terms of outputs and then we deal with the introduction of internal variables and then with their subsequent orientation. The problem of Model Orientation introduces a partitioning of the invariant structure of $H(s)$ polynomial model (Smith form structure and Forney invariants [52]) and leads to the definition of a family of models $\Sigma(T, V, U, W)$ with transfer functions having an invariant structure that has evolved from that of $H(s)$ [67].

Challenges 6.1 The study of algebraic structure of the $\Sigma(F, G)$ and $\Sigma(T, V, U, W)$ families as functions of selection of model orientation involve open issues such as:

- Use model orientation to parameterize all possible invariant structures for oriented state space and polynomial models.
- Explore the role of model orientation on the resulting system properties of the oriented models and investigate procedures leading to improved system properties, as a result of model orientation selection.
- Develop model orientation for constrained versions of the problems above.

■

6.2. Model Projection Problems

For many systems the number of potential control variables and potential measurements, which ideally may be used, can become very large. Developing criteria and techniques for selection of an effective input, output scheme, as projections of the extended input, output vectors respectively, is what we call*Model Projection Problems* (MPP). For linear systems, where orientation has already been decided, and represented by a progenitor rational matrix $G(s) \in \mathbb{R}(s)^{q\times p}$ the MPP is equivalent to selecting the sensor, actuator maps h, g (or $m \times q$, $p \times l$ constant matrices H, Q, $m \leq q$, $l \leq p$) such that the transfer function $W(s) = HG(s)Q$, $W(s) \in \mathbb{R}(s)^{m\times l}$ has certain desirable properties. Clearly, the problem as stated above is in the form of a generalised two parameter Model Matching. Note that the H, Q maps are not completely free, but they are constrained by the nature of the specific problem.

The types of problems considered here express a form of model structure evolution linked to the process of obtaining new models by reducing a larger original input, or output structure. In this sense, projection tends to aggregate, reduce an original model to a smaller dimension with desirable properties. A special problem within this area, that has been studied so far is the zero assignment problem by squaring down [10]-[14]. The selection of the effective input, output schemes is reduced to selection of type of Q, H matrices and using a variety of criteria. If the system is to be used for control purposes, control criteria have to be used for the assessment of effectiveness of the actuator, sensor schemes represented by the Q, H maps respectively. Some key issues expressing the model evolution in the process of selection of the effective sets of inputs, outputs are:

- **Desired Generic Dimensions Problem**: Defining the desirable general characteristics, such as desirable number of inputs, outputs on a system model, with reference to some assumed internal structure (graph, or generic model with an assumed McMillan degree [24]) is referred to as the desired generic dimensions problem. Such investigations are carried out on progenitor models and aim at using conditions, for generic solvability, or generic system properties to define the least required numbers of effective inputs, outputs needed for certain structural properties. Early results are based on indicators, such as the Segre index [70]. The use of existing results on generic solvability of control problems leads to constraint integer optimization problems aiming to produce generic families of systems for a which a range of problems may be solved. The solvability of control problems with different control schemes introduces alternative criteria which involve the McMillan degree, and/or the generic infinite zero structure. Use of such invariants requires the development of methodologies for their robust computation and this is what we refer to as structural identification [54] on early process models.

— **Input-Output Structure Reduction Problem (I-ORP) and Well Conditioning of Early Models**: Frequently, the resulting model from the process of model orientation has physical input, output variables, which is a desired property to preserve, but the overall system is not well-conditioned in terms of its properties. Then a special form of orientation model projection takes place, where only an α subset of inputs and a β subset of outputs is used, leading to an $S_{\alpha,\beta} = (A, B_\alpha), C_\beta, D_{\alpha,\beta})$ subsystem with corresponding transfer function $G_{\alpha,\beta}(s)$. The objective here is to select the α, β sets such that the resulting $S_{\alpha,\beta}$, $G_{\alpha,\beta}(s)$, is well structured as far as certain properties, which may include input, output regularity, non-degeneracy, minimality etc [71]. Such a problem will be referred to, in short, as *well conditioning by input-output reduction* (WCP). Note that in a transfer function matrix setup, WCP is equivalent to defining sub-matrices of $G(s)$ by eliminating certain columns and rows and which have desirable properties. An integral part of the study is the parameterisation of the maximal input, output cardinality solutions [71].

— **Graph Structural MPPs (GS-MPP)**: On early process design models of the graph type (state space formulation) or structured transfer function type, there is frequently the need to define subsets of inputs, outputs at the local, or global level, or appropriate structural combinations of them to guarantee structural properties such as controllability, observability, rejection of disturbances etc. Issues related to robustness under fault conditions may also be used as criteria here. In terms of the two parameter scheme associated with MPP, the problem here is to expand the tasks described in (b) by defining the required Boolean structure of Q, and H transformations on an internal model, or the modification to the internal graph that can guarantee such properties. This area involves the examination of the Kronecker structure of graph structured pencil models, which are linked to the presence, or absence of certain system properties, and the study of properties for systems with closely related graphs.

— **System Squaring Down and Structure Assignment Problems**: On linear progenitor models the selection of given dimension and possible structure constant Q or H, matrices, leads to new models where the invariant structure is obtained by appropriate transformation of the progenitor model invariant structure. Apart from the study of generic properties and their link to discrete type of invariants, there is also the need to investigate the effects of input, output reduction transformation on well defined models and specific parameters. The transformation of one set of invariants to another is a problem not fully understood; certain results in relationship to decoupling and in relation to the Morgan's problem are in [72], whereas in [16] it has been shown that the onesided MPPs are equivalent to generalised cover problems of geometric theory. A special case of IS-MPP is the zero assignment by squaring down [11]-[14], which is well developed and linked to the general formulation of *Determinantal Assignment Problems* [73]. The two parameter version of squaring down aims for a transfer function $W(s)$ which is square and has a given zero structure; similar problems may be defined for the case of graph structured state-space models instead of the transfer function formulations. For all such problems, the overall philosophy is to design Q, H such that the resulting model has a given desirable invariant structure or avoids having undesirable structural characteristics.

— **Performance Optimisation Model Projection Problems**: For a linear progenitor model $H(s)$, we may pose problems where we avoid formation of certain undesirable structural characteristics (such as right half plane zeros, high order infinite zeros etc) and at the same time optimise the values of certain key indicators, such as singular value properties of controllability, observability Grammians, condition number, etc. Within this class we may also consider the problems where the selection of Q, H aims at minimising some form of uncertainty of the progenitor model. The overall approach here is to utilise the degrees of freedom in Q, H matrices, which exist when avoidance of structural features rather than assignment of them is the central objective, to optimise certain key performance, or control structure indicator. The above cluster of problems define important areas of research for developing the system aspects of instrumentation.

Challenges 6.2 Specific problems in the selection of input and output structure, which are open are:

i) Define the lowest bounds for the number of effective inputs, outputs, which are needed for certain control scheme, or family of alternative control schemes.

ii) Define the best location of effective inputs, outputs, as well as, the structure of actuator, sensor maps, which may guarantee structural controllability, and observability.

iii) Evaluation of degree of dependence, independence of given input, output instrumentation schemes and its implications on process controllability, observability.

iv) Evaluation of effect of a selected input, output scheme on the control quality, characteristics of the final system and selection of "best" schemes for easy, reliable control.

v) Define the optimal sensor-actuator distribution to achieve certain control objectives on the resulting model.

6.3. Structure Re-interpretation Problems

The structure of a linear system is defined in terms of invariants (under certain types of transformation groups) and the underlying graph structure of the interconnection topology. Different descriptions may be associated with a system (implicit, regular, rational etc) and for a given system the relations between such descriptions may be expressed in terms of their structural invariants. Such relations may be seen as forms of re-organising the system structure. We distinguish two important classes of problems:

— The Feedback Simulation Problem, [74], [75]

— Proper Invariant Realizations of Autonomous Descriptions [76]

Feedback Simulation Problem: This class of problems involves the transformation of invariants from internal to external descriptions and may be expressed as follows: Consider the linear system of (1) and a strictly proper transfer function matrix $T_m(s) \in \mathbb{R}^{m\times q}[s]$, which is referred to as a model.

$$S(A,B) : \underline{\dot{x}}(t) = A\underline{x}(t) + B\underline{u}(t),\ A \in \mathbb{R}^{n\times n},\ B \in \mathbb{R}^{n\times r},\ rank(B) = r \tag{5}$$

Let us assume that there exist matrices $C \in \mathbb{R}^{m\times n}$, $G \in \mathbb{R}^{r\times q}$, $F \in \mathbb{R}^{r\times n}$ where $rank(C) = m$ and $rank(G) = q$ such that the following condition holds

$$T_{C,F,G}(s) = C(sI - A - BF)^{-1}BG = T_m(s) \tag{6}$$

Then we say that the feedback system given by $T_m(s)$ is simulated by the system $S(A,B)$, the matrix C and the state feedback law $\underline{u} = F\underline{x} + G\underline{v}$. This is referred to as Simulation Problem which is an extension of the Feedback Simulation [74], that provides conditions for the realization of proper rational matrices as state feedbacks, or output injection laws. The current version is more general than the classical version, since the introduction of C provides extra degrees of freedom. Forms of feedback which are non-regular as far as the input transformation G, change aspects of the structure; such transformations of invariants are significant in the study of the Morgan's problem (row-by-row decoupling problem) [72].

Proper Invariant Realizations: The third class of problems is linked to the interpretation of problems of implicit systems (geometric theory etc) to regular state space theory and vice-versa. This establishes an equivalence between the trajectories of the autonomous first order description $S(F,G) : F\underline{\dot{\xi}} + G\underline{\xi}$ and the forced solutions of the regular state space descriptions $S_r(\hat{A},\hat{B},\hat{C})$ referred to as proper invariant realizations of $S(F,G)$ [76]. This equivalence uses the explicit links between the Kronecker structures of the pencils associated with the two systems [76]. The Kronecker structure of $S_r(\hat{A},\hat{B},\hat{C})$ evolves from that of $S(F,G)$ in a prescribed manner for the different types of invariants.

6.4. Model Expansion Problems

Defining input test signals and corresponding output measurements, is an integral part of the identification, modelling exercise. Defining input output schemes with the aim to identify, (or improve) a system model, or reconstruct an unmeasured internal variable, is what we mean by *Model Expansion Problems* (MEP). Questions related to the nature of test signals, or properties of the measured signals are also important here, on top of the more general questions related to the structure of the i/o scheme; the latter gives a distinct signal processing flavour to MEP. Some distinct problem areas are:

— **Additional Measurements for Estimation of Variables**: Frequently in process control, some important variables are not available for measurement. Secondary measurements have to be selected and used in conjunction with estimators to infer the value of unmeasurable variables. The proper selection of second measurements is a task of paramount importance for the synthesis of control schemes. The various aspects for the problem are discussed within the area of state estimation.

- **Input, Output schemes for System identification**: The selection of input test signals and output measurements is an integral part of the setting up of model identification experiments. In fact, the identified model is always a function of the way the system if excited and observed, i.e., of the way the system is embedded in its experimental environmental. Most of the work so far has concentrated on SISO identification techniques and on the effect of test signal characteristics on the identification aspects of the model [77]. The study of effect of location of the group of excitation signals and corresponding group of extracted measurements on the identification problem has not been properly examined so far and its proper study is long overdue. Issues such as how and whether additional excitation signals and extracted measurements may enhance the scope and accuracy of identifiable models are challenging problems. This area of work is closely related to the problem of identifiability of models [77].

- **Model Completion Problems**: This class of problems, deals with the problem of augmentation of a system operator, like the matrix pencil and has a dual nature to that of model projection, since now we deal with dimensional expansion of the relevant operator. Such problems are defined as: Let $sE - H$ be an $r \times q$ pencil, which has a sub-pencil of the $(r + t) \times (q + v)$ pencil $sE' - H'$, where

  $$sE - H = \begin{bmatrix} sE' - H' & X \\ X & X \end{bmatrix} \tag{7}$$

  and the X's stand for unspecified pencils of compatible dimensions. It is of interest to study the relationships between the sets of invariants of $sE - H$ and $sE' - H'$ pencils and in particular examine the conditions under which we may assign arbitrarily the structure of $sE - H$. This problem is known as *Matrix Pencil Completion Problem* (MPCP) [78],[79] and includes as special case the problem of invariant polynomial assignment by state feedback [80]. The above formulation may be also extended to that of expansion of rational models.

Model expansion are once more examples of model structure evolution, where additional inputs, outputs help a system model to grow to a more full representation of the existing system. The problems in this area express an alternative form of evolution of structure and properties of the model by manipulation of the input-output, external structure.

7. Physical Growth-Death System Evolution and Re-engineering

7.1. Introduction

A new and challenging class of problems that expresses a form of system evolution emerge within the framework of reengineering of technological processes. A special case that is simpler and allows the development of the basic principles is the re-engineering of passive electrical network that is used as a prototype of redesign of general systems. Such problems aim to alter the dynamic behaviour by changing the system structure as well as nature and values of system components, without resorting to design of a controller. Although, such problems are not traditional control problems, their study relies on the deployment of control techniques. Systems re-engineering introduces new types of systems evolution and it is achieved by a number of actions distinct from control design aiming to change the overall system behavior. These actions involve:

- Changing the values of the components of the system
- Altering the nature of components without changing the topology
- Modifying the topology and possibly reducing the system by removing components/subsystems.
- Augmenting the system by adding subsystems to the existing topology.

The problems we are addressing here are questions of open-loop design of the system on which control is eventually applied. Such problems are distinct from control, but their study involves system and control concepts and their formulation may be reduced to problems that may be identified as problems of the structural system framework; in particular, some of these problems may be formulated as problems of frequency assignment [73]. A major challenge in this study is the fact that the evolutionary transformations cannot be studied in a convenient unifying manner with the traditional state space, or transfer function representations. In fact, every time we perform transformations such as changing values for dynamic or non-dynamic elements within a fixed interconnection topology and/or alteration of the network interconnection topology and possible increase of elements, branches, new state space or transfer function models have to be worked out. Defining an appropriate representation that can facilitate the study of these structural, physical, or parametric changes leads us to consider the traditional network theory models provided by the Impedance and Admittance operators [81],[83]. These operators allow the uniform representation of changes of a many dynamic, or non-dynamic elements with preservation, or alteration of existing topologies without, or with changes in the overall nodal or loop cardinality of the network.

7.2. **Impedance and Admittance Models**

Classical network theory is based on models derived based on:

- Network loop analysis
- Network node analysis

In the network loop analysis method, the variables are selected such that the vertex law is automatically satisfied. We consider only planar graphs with b branches and n vertices and we define the variables associated with each of the meshes, which are referred to as *loop* variables. The path law is then written for each mesh and substitutions are made for the across variables in terms of the loop variables using the elemental equations. The overall system is reduced to a number of meshes, which are $q = (b - n + 1)$ [83] and will be referred to as *loop cardinality* of the network. If we denote by $(f_1, \ldots, f_q)$ the set of the Laplace transforms of the loop currents and by $(u_{s1}, \ldots, u_{sq})$ the set of Laplace transforms of equivalent voltage sources, then the loop or impedance model is defined by [84], [83]:

$$Z(s)\underline{f}(s) = \underline{u}_s(s) \tag{1}$$

where, $Z(s)$ has elements $z_{ii}(s)$ expressing the sum of impedances in loop i and $z_{ij}(s)$ is the sum of impedances common between loops i and j. This is referred to as the loop or *impedance model* and it is an integral-differential symmetric matrix and $Z(s)$ is referred to as the network impedance matrix.

Alternatively, we can use the across variables from each vertex to some reference vertex are chosen as the unknowns, called *node variables*, and in terms of which the final set of equations is formulated. These variables automatically satisfy the path laws and the vertex equations are written at each node, whereas the through variables are then expressed directly in terms of the node variables. The process eliminates all variables except the node variables and has a number of equations, which is in general $p = (n-1)$ and will be referred to as *nodal cardinality* of the network. The node method is the dual to the loop method and the basic steps involve the selection of internal nodes and corresponding node voltages and the transformation of the voltage sources to equivalent current sources (Norton's theorem). If we denote by $(u_1, \ldots, u_p)$ the Laplace transforms of the reduced node voltages and by $(i_{s1}, \ldots, i_{sp})$ the set of Laplace transforms of equivalent current sources, then the node or admittance model is defined by [84],[83]:

$$Y(s)\underline{u}(s) = \underline{i}_s(s) \tag{2}$$

where, $y_{ii}(s)$ is the sum of admittances in node i and $y_{ij}(s)$ is the sum of admittances common between nodes i and j. This is referred to as the node or admittance model and it is an integral-differential symmetric matrix and $Z(s)$ is referred to as the network admittance matrix.

7.3. **The Autonomous Natural Impedance-Admittance Model and Topologies**

When we consider networks with no inputs (no current, or voltage sources) the resulting admittance, or impedance network models may be described in a unifying way as:

$$S(B, C, D) : \left\{pB + p^{-1}C + D\right\} \underline{x}(t) = 0 \tag{3}$$

where p, p^{-1} are respectively the differential, integral operators respectively and is the vector of nodal voltages, or loop currents. Such a description is referred to as the natural autonomous network description [81] and the operator $W(s) = sB + s^{-1}C + D$ is called the natural network operator. Note that for the case of admittance we have that B is a matrix of $A-$type elements (i.e. mass, inertance, capacitance), C is the matrix of $T-$type elements (i.e. spring, inductance) and D is a matrix of $D-$type elements (i.e. resistance, frictions). For the case of impedance the reverse holds true. Hence, B is the matrix of $T-$type elements, C is the matrix of $A-$type elements and D is the matrix of $D-$type elements. The symmetric operator $W(s)$ is thus a common description of $Y(s)$ and $Z(s)$ matrices. The operator $W(s)$ describes the dynamics of the network and of special interest are the properties of its zeros.

The description (3) has no inputs and no outputs and as such is a new type of implicit description that corresponds to loop or node and may be referred to as network implicit description. If inputs and outputs are introduced, then we obtain an oriented description

$$S(B, C, D; H.F) : \left\{pB + p^{-1}C + D\right\} \underline{x}(t) = H\underline{u}(t),\ \underline{y}(t) = F\underline{x}(t) \tag{4}$$

with a transfer function matrix $G(s) = FW(s)^{-1}H$. Clearly, the selection of the pair (H, F) defines a family of new oriented models $S(B, C, D; H, F)$ and their structural properties (graph and invariant) are evolutions of those of $S(B, C, D)$. Assigning the invariant structure of the oriented model as an evolution of the implicit model introduces many open challenges for structural control methodologies. The importance of $S(B, C, D)$ and $S(B, C, D; H, F)$ descriptions is that there is a well identified graph structure associated with these descriptions. Network modeling uses the system graph, G_s which is the basic topological structure that generates the system equations. Apart from the system graph we may introduce some additional topologies, which are linked to the specifics of the Node and Loop analysis.

The Vertex Topology: Every network may be represented in terms of a set of vertices, or nodes and all branches between two vertices may be represented by an admittance function. Specification of the values of the across variables of the vertices defines the values of all through variables in the network. The nature of the elements in the branches of the natural vertex graph defines an element dependent topology, which is characterized by adjacency type matrices. If we set the external sources to zero, the reduced graph will be referred to as the kernel vertex graph G_v. For a given G_v we define the corresponding $A-$, $T-$, $D-$ sub-graphs $G_{v,a}$, $G_{v,t}$, $G_{v,d}$ by eliminating from the G_v the elements (T, D), (A, D), (A, T) respectively. By eliminating all A, T, D elements we get the sources sub-graph $G_{v,s}$ representing the location of the through variable sources. The latter define a decomposition of G_v, denoted by:

$$G_v = G_{v,a} \cup G_{v,t} \cup G_{v,d} \cup G_{v,s} \tag{5}$$

If $(A_{v,a}; A_{v,t}; A_{v,d}; A_{v,s})$ are the adjacency matrices of the above sub-graphs then these provide a representation of the vertex topology of the network.

The loop topology: The loop topology is a notion dual to that of the vertex topology and it is based on the following principle: Every network of n vertices and b edges may be represented by $q = (b - n + 1)$ loops leading to independent equations. All branches common between two loops may be represented by an impedance function. Specification of the values of through variables for the loops defines the values of all across variables in the network. In a similar way to the case of nodal analysis, we may define the loop topology based on the kernel loop graph G_l and its sub-graphs, as well as a similar decomposition to that of (4) [84].

Challenges 7.1 Network modelling opens up new questions such as:

— Investigate the links between the G_v, G_l and G_s.

— Determine the degree of the zero polynomial of $W(s)$ in terms of the properties of G_v, G_l and G_s graphs.

— Examine the process of structural evolution from $W(s)$ to $G(s)$ as a function of the pair (H, F).

■

7.4. **Network Transformations**

The general classes of structural transformations which may preserve, or alter the cardinality of the network, and may also change its different types of topology; as well values and nature of elements are expressed as transformations on the operator $W(s)$ and are defined below:

Type 1: Changing the values of the components of the system without changing the topology as this is described by B, C, D tipple.
Type 2: Altering the nature of components by transformations on B, C, D tipple without changing the element cardinality of the network.

Type 3: Modifying the network's topology and changing the cardinality of elements by removing components/subsystems.
Type 4: Augmenting the network's topology and changing the cardinality of elements of the system by adding subsystems to the existing topology of the network.

We consider first Type 1, 2 transformations preserving the loop, or nodal cardinality and thus the dimensionality of B, C, D. The structural transformations are then expressed as [82], [84]:

Network Cardinality Preserving Structural Transformations: Given the triple of matrices B, C, D we consider transformations on the network matrices of the type

$$C' = C \pm c(x, b),\ B' = B \pm l(x, b),\ D' = D \pm r(x, b) \tag{6}$$

which preserve the physical elements cardinality (loop, or nodal cardinality) and depend on the real parameter x and the position vector b . The changes $c(x, b)$, $l(x, b)$, $r(x, b)$ have the general form $f(x, b)$ [82],[84] where:

$$f(x, \underline{b}) = x\underline{b} \cdot \underline{b}^t,\ \underline{b} = \underline{e}_i \ or\ \underline{b} = \underline{e}_i - \underline{e}_j, i \neq j \tag{7}$$

■

Clearly, the above single parameter changes in values and topology under the fixed cardinality assumption may be extended to multi-parameter changes due to the additive property and thus the new triple of matrices may be described as

$$\begin{aligned} C' &= C \pm \{c(x_i, b_i), i \in v\} = C + C^* \\ B' &= B \pm \{l(x_i, b_i), i \in v\} = B + B^* \\ D' &= D \pm \{r(x_i, b_i), i \in v\} = D + D^* \end{aligned} \tag{8}$$

Clearly, the multi-parameter changes expressed by the triple B*, C*, D* affect the resulting structure of S(B′,C′,D′) and S(B′,C′,D′; H,F) systems. The zero assignment of the single parameter case has been considered in [82], [45] and the multi-parameter case in [84]-[86]. A number of open challenges are:

Challenges 7.2 Network redesign opens up new questions such as:

- Investigate the solvability of zero assignment of $W(s)$ using as design parameters the structured triple B^*, C^*, D^*.
- Examine the process of structural evolution of $S(B, C, D; H, F)$ (evolution of invariant structure) as a function of the structured triple B^*, C^*, D^*.

■

Network Cardinality Altering Transformations: We distinguish the case where a set of loops or notes are lost, or removed and the case where a set of loops, or nodes are added. If $W(s)$ is the natural network operator (impedance, or admittance). Then we distinguish:

Network Cardinality decrease: If we rearrange the order of nodes or loops such that it is the last σ set that is removed we can express the reduced $W^*(s)$ as the submatrix obtained from $W(s)$ by deleting the last σ set of rows and columns (parts $X(s)$ and $X'(s)$) of $W(s)$ ie:

$$W(s) = \begin{bmatrix} W^*(s) & X(s) \\ X(s) & X'(s) \end{bmatrix} \tag{9}$$

Network Cardinality increase: If it is assumed that it is the last σ set of loops, or nodes that is added (addition of $X(s)$ and $X'(s)$ matrices) we can express the expanded matrix $W^+(s)$ as

$$W^+(s) = \begin{bmatrix} W(s) & X(s) \\ X(s) & X'(s) \end{bmatrix} \tag{10}$$

■

Clearly, such transformations express an evolution on $S(B, C, D)$ and $S(B, C, D; H, F)$ network descriptions and this opens new challenges:

Challenges 7.3 Network redesign by elimination of nodes/loops or augmentation by addition of nodes/loops opens up new questions such as:

- Investigate the zero structure properties of $W^*(s)$ under node/loop elimination and the properties of $W^+(s)$ under node/loop addition [45].
- Examine the process of structural evolution of $S^*(B^*, C^*, D^*; H, F)$ and $S^+(B^+, C^+, D^+; H, F)$ (evolution of invariant structure) as a function of the structured deletion, or augmentation transformations.

■

8. Operational Complexity and System of Systems

A significant new form of systems complexity arises when a holistic view for the system is considered where apart from the physical (infrastructure) dimension we also consider the Operations and Business together with the related activities of Data, Signals and Communications. The integration of all these dimensions introduces a new form of complexity that is frequently referred to as System of Systems (SoS) [5],[6] complexity. This new type of complexity introduces an evolution of the traditional Graph interconnection topology to a new form referred as "systems play" [7],[36].

8.1. Integration of System Operations, Data, Signals and Business

The integration of Operations, Signals, Data and Business is an integral part of the "*Integrated Manufacturing System*" [1] paradigm and this introduces many features of the new notion of "*System of Systems*" (SoS) which deviates from the traditional engineering paradigms, mostly based on the physical system. The overall integrated manufacturing system has aspects which may be represented by the diagram of Figure (7), where activities are grouped in the four main areas described by the:

(i) Physical Process Dimension

(ii) Signals, Operations Dimension

(iii) Data, IT, Software Dimension

(iv) Organisational Dimension

which they interact strongly between themselves. Such interactions lead to features associated with the family of System of Systems.

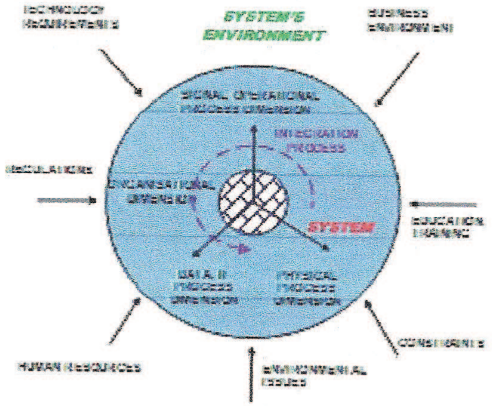

Figure 7: Basic System Shell of Manufacturing Integration

The Physical Process Dimension deals with the physical process itself and it is the traditional view of the Engineering Process. The Signals, Operations Dimension is concerned with the study of the different operations, functions based on the Physical Process and it is thus closely related to operations for production. The extensive use of IT has increased considerably the interaction between these processes and the physical system. Signals and information extracted from the process are central to the problem of integration and understanding the connectivities between the alternative operations, functionalities is crucial for controlling the overall system behaviour. Both design and operations generate rely on data and deployment of software tools for processing of data introduces additional systems, which are strongly linked to the management of the different processes. There are different ways of organising the interaction of the different functionalities and their IT representations with the physical process and this expresses the organisational dimension of the overall system. The latter represents a way of linking processes and expresses a higher level of “connection topology” that is referred to as complex system organisation. An alternative, thematic simple illustration of the overall industrial enterprise level activity is given in Figure (8) which indicates a natural nesting of problem areas, where the physical system and the corresponding design issues provide the core, linked with the formation of the physical process that realises production.

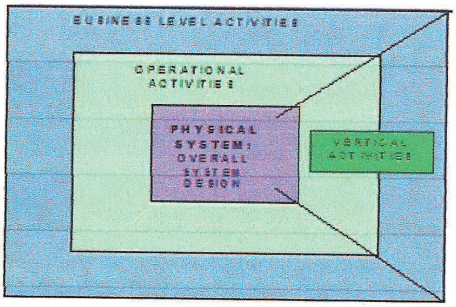

Figure 8: Nesting of Industrial Enterprise level activities

Production level activities are usually organised in a hierarchical manner and they realise the higher level strategies decided at the business level. However, hierarchical organisation is not the only possible and alternative organisation forms [35] induce different topologies in the information/data structures. Vertical activities link the Business-Operations-Design hierarchy. Integration of Business tasks, Operational issues and Design aspects introduces new challenges. These involve quantifying the relationships between problems on a horizontal (same level), as well as vertical (going through different levels) directions, and introduce new forms of systems complexity manifested as aspects of the family of SoS. SoS features associated with this problem include:

- Extension of the standard notion of interconnection (graph) topology to the new notion of systems play [36].
- Variability of the organizational information/control structure to respond to changing operational goals.
- Uncertainty on the definition of clear system boundaries at the subsystem level.
- Multilevel coupling of physical subsystems and operational processes at horizontal and vertical directions leading to multilevel decision and diagnostics problems.

The above challenges introduce new open problems discussed later on in the area of System of Systems (SoS).

8.2. **Traditional Notion of System of Systems**

The term system of systems (SoS) has been used in the literature in different ways [5]-[7]. Most definitions ([5],[6]) describe features or properties of complex systems linked to specific examples. SoS typically exhibit aspects of the behaviour met in complex systems, but not all complex problems fall in the realm of SoS. Problem areas characterized as SoS exhibit features such as [5]: Operational Independence of Elements; Managerial Independence of Elements; Evolutionary Development; Emergent Behaviour; Geographical Distribution of Elements; Inter-disciplinary Study; Heterogeneity of Systems; etc. The definitions that have been given so far [5],[6],[88] contain elements of what the abstract notion should have, but they are more linked to specific features of areas of applications. A more generic definition that captures the key features and which is a good basis for further development is given below [5]:

Definition 8.1 *(i)* Systems of systems are large-scale integrated systems which are heterogeneous and independently operable on their own, but are networked together for a common goal. The goal, as mentioned before, may be cost, performance, robustness, etc.
(ii) A System of Systems is a "super system" comprised of other elements which themselves are independent complex operational systems and interact among themselves to achieve a common goal. Each element of a SoS achieves well-substantiated goals even if they are detached from the SoS.

■

The above definitions are descriptive and they capture crucial features but they do not answer the question, why the new notion is different from that of composite systems. The distinctive feature of our approach is that we treat the notion of *System of Systems* (SoS) as an evolution of the standard notion in engineering of *Composite Systems* (CoS) [13]. We note:

- Both CoS and SoS are compositions of simpler objects, or systems.
- Both CoS and SoS are embedded in the environment of a larger system.
- The objects, or sub-systems in CoS do not have their independent goal, they are not autonomous and their behaviour is subject to the rules of the interconnection topology.
- The interconnection rule in CoS is expressed as a graph topology.
- The subsystems in SoS may have their own goals and some of them may be autonomous, semi-autonomous, or organised as autonomous groupings of composite systems.
- There may be a connection rule expressed as a graph topology for the information structures of subsystems.
- The SoS has associated with it a global operational task where every subsystem enters as an agent with their individual Operational Set, Goals.

8.3. A New Characterisation for the System of Systems

Developing a generic definition for SoS that transcends specific domains of applications is essential for the characterization of this new family and the development of an appropriate systems engineering framework that covers all challenging new features. Note, that the system appears as an autonomous agent (internal system structure together with its inputs and outputs), having its operational instructions and goals and a pair of information vectors expressed by the input and output influences vectors. Additional properties may be introduced by assuming that the system under consideration has the control, modelling and supervisory capabilities integrated within it which enable the system to act as an agent with independence capabilities and act as a player in games. We may represent such systems as illustrated in Figure (9) and refer to them as integrated systems. The latter term is used to distinguish it from systems which have no integrated control and information processing capabilities and which may be referred to as simple systems. If such a system is embedded in a larger system (Composite, or System of Systems) relations with other systems may be defined in two different ways:

(i) An interconnection topology of the graph type defined on the set of input-, output- influences subsystem information structures.

(ii) A global game where every subsystem enters as an agent with their individual Operational Set, Goals.

The distinguishing feature of the SoS is that the subsystems participate in the composition as intelligent agents with a relative autonomy and may act as players in a game. The latter property requires that the systems entering the composition are of the integrated type, since this requires capabilities for control, estimation, modelling and supervisory capabilities. Features, such as large dimensionality, heterogeneity, network structure, Operational, Adaptability, Emergent Behavior etc may be also present in the case of CoS as well. We define:

Definition 8.2 Consider a set of systems $\Sigma = \{S_i, i = 1, 2, \ldots \mu\}$ and let F be an interconnection rule defined on the information structures of S_i systems. The action of F on Σ, called a *Composite System*, or the composition of Σ under F.

■

The information structure of each system is defined by the pair of the input and output influence vectors and the interconnection rule may be represented by a graph topology. The resulted system is embedded in a larger system and it is treated as new system with its own system boundary. In the above definition the systems considered are simple and not necessarily integrated. This definition may now be extended as follows:

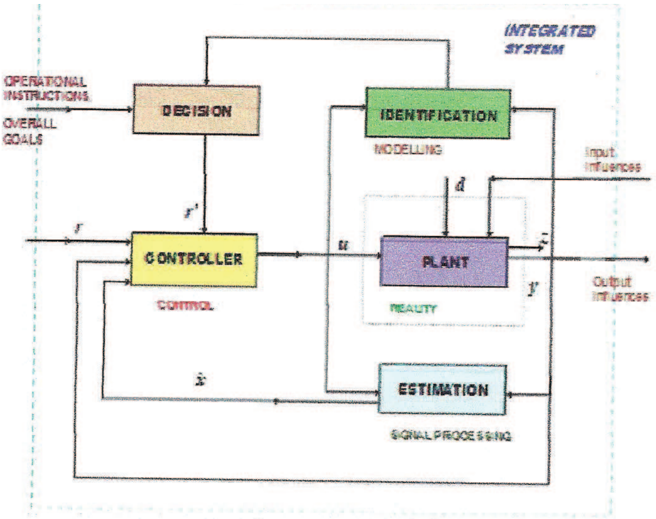

Figure 9: Representation of an Integrated System

Definition 8.3 Consider a set of integrated systems $\Sigma = \{S_i, i = 1, 2, \ldots \mu\}$, F be an interconnection rule defined on the information structures of S_i systems and let $S_c = \Sigma^* F$ be the resulting composite system. If G is a general rule of operations, referred to as "*systems play*" that is defined on the systems i then the action of G on S_c is a new system $S_c^* = \Sigma^* F \bullet G$ which will be called a *System of Systems*, or the, F, G composition of Σ.

■

The notion of SoS thus emerges as an evolution of CoS since the systems are assumed to be integrated, ie having capabilities for information processing and thus they are capable to act as agents and participate in games of some type. Note that the transition from the CoS to SoS involves moving from simple to integrated systems as far as the subsystems, and the introduction of the new notion of "systems play" which emerges as a generalization of the notion of topological composition. The nature of the applications defines the systems play, which frequently may be expressed as a game. If not all subsystems are integrated we may define:

Definition 8.4 Consider a set of systems $\Sigma = \{S_i, i = 1, 2, \ldots, \rho; S_i', i = 1, 2, \ldots, \sigma\}$, where the $S_i, i = 1, 2, \ldots, \rho$ subset is integrated and the $S_i', i = 1, 2, \ldots, \sigma$ subset is simple. We consider to be an interconnection rule defined on the information structures of sub-systems of Σ and let $S_c = \Sigma^* F$ be the resulting composite system. IfG is a *systems play* defined on the integrated systems S_i then the action of G on S_c

is a new system $S''_c = \Sigma^* F \bullet G$ which will be called a *Weak System of Systems*, or the weak F, G composition of Σ.

■

The essence of the new definition is that SoS emerges as a two dimensional notion. At the lower level it appears as a composite system with some interconnection topology defined on the subsystems, which are now assumed to possess information processing capabilities. The latter property allows these subsystems to act as agents and SoS to emerge as a multi-agent system (MAS) composed of multiple interacting intelligent agents. This multi-agent systems view allows SoS to act as vehicle to solve problems which are difficult or impossible for an individual agent. The multi-agent dimension of SoS allows SoS to develop "self-organization" capabilities and has important characteristics such as [87]:

- Autonomy: the agents are at least partially autonomous
- Local views: no agent has a full global view of the system, or the system is too complex for an agent to make practical use of such knowledge
- Decentralization: there is no designated single controlling agent, but decision and information gathering is distributed.

8.4. **Classification of System of Systems**

The major challenge in the development of a unifying approach to the study of SoS is the quantitative characterisation of the new notion of the systems play. Taking into account that SoS problems emerge in many and diverse domains, it is clear that some classification of the general SoS family into sub-families with common characteristics is essential before we embark to the characterization of notions such as systems play and subsequently address the study of emergent properties for such systems. We may classify SoS according to different criteria such as the nature of the origin:

(i) Physical, or natural SoS (N-SoS)

(ii) Engineered or Constructed SoS (E-SoS)

The first category involves problems of the natural world, and social-economic problems and are the results of evolution of physical, or socio-economic processes. Problems such as the "ecosystem" of a geographical region, and issues such as "social plenomena" are typical examples. The common characteristic of these classes is that they are the results of a "natural evolution" and they are not the by-products of some notion of design. Of course, there are grey areas between the two classes such is the case "global economy" where evolution is accompanied by some effort to intervene and affect the economic processes. It is important to note that in N-SoS some "goals", "principles" drive the development of the system play, whereas in E-SoS the "goal" is driven some coordination effort. The relations between the subsystems provides another way for classification:

(a) Goal Driven and Unstructured (GU-SoS)

(b) Goal Driven with Central Coordination (GC-SoS)

In GU-SoS class the central goal for the system operation is set, as well as the environment within which the system operations will take place. In this case the nature of the system play is entirely defined by the set goal. In such cases the goal may define a form of a game where the intelligent agents may participate. Typical examples are problems related to “eco-systems”.

The GC-SoS class has the same features as the P-GU-SoS and similar subclasses with the additional feature the existence of coordination. The presence of coordination imposes a structure to the interpretation of the goal by the subsystem and the development of appropriate scenarios to achieve the central goal and partial goal. The nature of coordination also introduces special features to SoS characterization since it introduces a structure to the resulted systems play. Coordination is a form of organization and there may be different types such as “Hierarchical”, “Heterarchical” and “Holonic” [35]. Such forms of organization structure the systems play and the development of scenarios. Man-made systems usually involve coordination which drives the development of the system play. Further classification may be introduced by the nature of the origins of SoS. Types of SoS where the subsystems are of the engineering type without human action involvement are referred to as “hard” and those involving human behaviour are referred to as “soft”.

8.5. **Potential Methodologies for Analysis of System of Systems**

The system-wide coordination of real-world systems of systems is a challenging and open problem. The development of a description for the systems play depends on the nature of the particular SoS. In the following we outline different methodologies which may provide the required framework for describing systems play and thus assisting the development of SoS methodologies for different families of SoS by providing formal descriptions of the notion of systems play. Such methods are those referred to as Co-Operative Control, Market Based Coordination, Population Control Methodologies, and Coalition Games.

Co-Operative Control: The notion of Co-Operative Control has been used in a number of ways in the literature. A typical case describing a class of SoS very close to technological problems is the Vehicle Formation Problem [89],[90] defined as the control of the formation of k vehicles that are performing a shared task. It is assumed that the vehicles are able to communicate with the other vehicles in carrying out the task and they have capabilities to control their position in the effort to perform the task. Each vehicle is described as a rigid body moving in space and a state vector x_i may be associated with each one; by $x = (x_1, \dots, x_N)$ we may represent the complete state for the set of N vehicles. The collection of all individual states defines the state of the system and the execution of the assigned task requires the assignment of additional states that can make the system an SoS. The development of the scenario, task is handled by introducing for each vehicle an additional discrete state, α_i , which defines the role of the vehicle in the task and this is represented as an element of a discrete set $\mathcal{A}$. The definition of $\mathcal{A}$ depends on the specific cooperative control problem.

It is assumed that the vehicles are able to communicate with some set of other vehicles and the set of possible communication channels is represented by a graph g. The nodes of the graph represent the individual vehicles and a directed edge between two nodes represents the ability of a vehicle to receive information from another vehicle. Given a collection of vehicles with state $\underline{x}$ and roles $\underline{\alpha}$, we may define a task or scenario in terms of a performance function | the optimization of which is equivalent to the completion of the task. Clearly, such problems may also have constraints which make the problem a constrained optimization problem. For SoS the problems of interest are those involving cooperative tasks that can be solved using a decentralized strategy.

Market-Economics Based Coordination Techniques: The distinguishing feature of SoS is that there are autonomous units with their own management and control functions that are coupled by resource flows which need to be balanced, over appropriate periods of time depending local or global storage capacities. The performance of the subsystem consumption and production is influenced by availability of these resources [27]. To perform an arbitration of these flows requires economic balancing mechanisms [20],[21]. The management of the resource flows may be expressed as a network management problem, given that the resource flows define some generic network structure within which we define the flows. Clearly, the overall system performance and behaviour is influenced by discrete decisions taken. Two different approaches that can be used for the management of such flow-coupled SoS are: economics-driven coordination and market-based mechanisms. In both cases, the coordinator has only limited information about the behaviour and the constraints of the local units which perform a local optimization of their operational policies.

In the economics-driven coordination, it is assumed that the control of SoS involves the setting of production/consumption constraints or references between the global SoS coordinator and the controllers of individual systems. The SoS coordinator utilizes simplified models of the sub-systems, and a model of the connecting networks to compute references or constraints on the exchanged flows. The resulting optimization is based on the dynamic price profiles for the resources that are consumed or produced by the subsystems over the planning horizon. An alternative approach is to use mechanisms employing the concepts of economic markets to distribute limited resources between subsystems. The market is defined as a population of agents consisting of producers selling goods and consumers buying these goods [20], where the consumers' demand depends on the usefulness or *utility* of a good for the completion of its task. The goal of a market-based coordination mechanism is to generate equilibrium between the producers and the consumers such that the overall supply equals the overall demand. A popular mechanism to compute such equilibria is an auction and many different kinds of auction mechanisms have been developed [91]. Market-based mechanisms are inherently decentralized and can thus be mapped directly to systems with autonomous subsystems.

Population control methods: Population control refers to systems that comprise a large number of semi-independent subsystems, which macroscopically are viewed in terms of their emergent behaviour. Such systems are used in ecology to capture the fluctuations in the populations of interacting species and the relevant models use continuous variables to capture populations and differential equations to capture their evolution. There are extensions to hybrid models [95] and to delay and/or stochastic differential equation models. Of special interest is the class of *mixed-effect models* [94], which address the evolution of a heterogeneous population of individuals, which deploy ordinary differential equations, but with parameters linked to appropriate probability distributions. Population systems dynamics are gaining in importance, as man-made systems become increasingly complex and larger-scale and control of the emergent behaviour of large collections of semi-autonomous subsystems becomes an issue. Such methods are primarily motivated by biological applications, but have potential for the engineering field of SoS. These methods need to be adapted and extended, if they are to be made applicable to engineered SoS.

Coalition Games: The basic idea of SoS is to consider the overall system as a set of subsystems that are controlled by local controllers or agents which may exchange information and cooperate. This feature demonstrates the link of SoS to distributed and decentralized control schemes with the additional property that the interaction between the subsystems may indicate a time-varying coupling. It is this special feature that indicates the links to a rather new category of management and control schemes referred to as coalitional management schemes [93]. In this paradigm different agents cooperate when there is enough interaction between the controlled systems and they work in a decentralized fashion when there is little interaction. A coalition is a temporary alliance or partnering of groups in order to achieve a common purpose or to engage in a joint activity [96]. A coalition of systems is a temporary system of systems built to achieve a common objective. Coalition building is the process by which parties come together to form a coalition. Forming coalitions requires that the groups have similar values, interests, and goals which may allow members to combine their resources and become more powerful than when they each acted alone.

Challenges 8.1 Transforming current practice on SoS by developing a generic theory and design methodologies involves a large number of challenges centered around the formal characterisation of systems play. Some of the key problems addressed are:

- Classification of the broad family of SoS into well defined subfamilies characterised by common features such as a common characterisation of the systems play notion.
- Identify families of systems play and adapt methodologies developed in different domains to characterise the systems play notion.
- Examine effects of system organisation, strength of connectivity of sub-systems and uncertainty in drawing the system boundaries in the classification of SoS families.
- Develop methods to characterise and measure emergent system properties [97].

■

9. Conclusions: Control Theory in the Context Systems Evolution

Control Theory and Design have developed around the classical servomechanism paradigm. New Challenging areas of applications have introduced new systems paradigms, which include the area of Systems Integration for large Complex Systems and he paradigm of System of Systems. Common between those two paradigms is the notion of *systems evolution* which centres around the evolution of the system and its fundamental aspects of its structure such as that expressed by the interconnection topology in shaping the system internal model, the selection of inputs and outputs and thus the coupling of the system with its environment and the respective implications on system invariants and finally the evolution of the interconnection topology to the notion of systems play characterising SoS. Many new challenges have been identified and a number of new systems and control theory problems have been defined requiring development of a number of new mathematical problems which are crucial for the development of systems and control theory. The two major new system paradigms emerging are:

- The family of Structure Evolving Systems (SES) [4]
- The family of System of Systems [5],[7],[36]

The overall philosophy, which is adopted in the SES case is that each particular design stage, in the overall design process, shapes a local model; the structure of this local model has important implications on what can be achieved at the next design stage, and it thus determines overall cost, operability, safety and performance of the final process. Each design stage starts with a model and decisions taken there contribute to the gradual shaping of the final structural characteristics; however, this happens within a range of possible options. Structural properties and thus performance, operability, etc. characteristics evolve, but not in a simple manner. This evolution of structure and related potential for delivering certain level of performance are not well understood. We would like to drive the model evolution along paths avoiding the formation of undesirable structural features and where possible to assign desirable characteristics and values. The development of SES methodology has major implications for control and systems design and specific analysis and design issues are discussed in [4]. Such developments are crucial in controlling the development of the Evolutionary Design Process and thus development of Control Theory and Design for Evolutionary Systems.

The new definition for the SoS is the starting point for the development of methodology that may lead to a better understanding the nature of SoS and in particular the characterization of emergent properties. Examining the rules of composition of the subsystems and their coordination as agents in a larger system defines a challenging new area for research and requires links across many disciplines. Examining in detail the special features of the different classes of SoS is crucial in the effort to provide a quantitative formulation of the notion of *systems play* which may take different forms in the different classes. This is also crucial in quantifying the notion of emergence in the SoS context. The potential for applications is well beyond the traditional engineering field, when powerful modeling tools are defined that may allow the study of design and decision problems of the respective classes of SoS. It is worth mentioning at this point that the majority of SoS are products of *physical*, or *technological* evolution, rather than products of systematic design and understanding evolutionary processes leading to the formation of SoS is crucial.
The development of the theory for SES and SoS requires respective developments in areas of Mathematics and Computer Science. Graph Theory, Theory of Algebraic Invariants, Linear Algebra, Computational Algebraic Geometry, Numerical Methods, Optimization Theory and Formal Methods are amongst the branches of Mathematics where new mathematical challenges have to be addressed.

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