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1. Let $q_1, q_2, \ldots$ be an enumeration of the set $\mathbb{Q} \cap [0,1]$. We define the set

$$E = \bigcup_{n=1}^{\infty} (q_n - \frac{1}{10^n}, q_n + \frac{1}{10^n}).$$

Show that the set $[0,1] \setminus E$ is not empty.

2. We define the set $A \subseteq [0,1]$ as follows. We define $A_0 = [0,1]$ and, for $n = 1, 2, 3, \ldots$, we define the set A_n to be what is left from the set A_{n-1} if from each one of the intervals that make up A_{n-1} we remove the middle 1/5 of the interval. Having defined the sets A_n we finally define $A = \bigcap_{n=0}^{\infty} A_n$. Show that $m(A) = 0$.

3. We say that a set $S \subseteq \mathbb{R}$ is of type G_δ if it is the countable intersection of open sets, i.e. if there exist open sets $G_n \subseteq \mathbb{R}$ such that $S = \bigcap_n G_n$. If $E \subseteq \mathbb{R}$ show that there exists a G_δ set $S \supseteq E$ such that $m(S \setminus E) = 0$.