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1. Consider the function $f(z) = z^2$. Describe the image of the region

$$\left\{ z \in \mathbb{C} : \ \frac{1}{2} \le |z| \le 2, \ \mathrm{Arg} z \in [\pi/4, \pi/2] \right\}.$$

Do the same for the function $g(z) = z^3$. Prove that the function $z \to \sqrt{z}$ can be defined in this region as a continuous function and find the image of this region through this function.

2. Using the Cauchy-Riemann equations investigate which of the following functions are differentiable and at which points of their domain of definition.

$$f(z) = \overline{z}, \ f(z) = z - \overline{z}, \ f(z) = 2x + ixy^2, \ f(z) = e^x e^{-iy},$$
$$f(z) = e^{-x} e^{-iy}, \ f(z) = (x+iy)^3, \ f(z) = \tfrac{1}{x+iy}, \ f(z) = x^2 + y^2, \ (x+iy)y.$$