All curves are positively oriented unless otherwise noted.

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1. If f is analytic at z_0 and $f'(z_0) \neq 0$ show that there exists $r > 0$ such that f is 1-1 on the set $|z - z_0| < r$.

2. Find Möbius transformations that map the points $0, 1, \infty$ to the points

$$(a)\ 0, i, \infty, \quad (b)\ 0, 1, 2 \quad (c)\ -i, \infty, 1 \quad (d)\ -1, \infty, 1.$$

3. Find a Möbius transformation that maps the half-plane $\mathrm{Re}\, z - \mathrm{Im}\, z < 1$ to the disk $|w| < 1$.

4. Find a conformal map $w = f(z)$ which maps the domain $0 < \mathrm{Arg} z < \frac{\pi}{4}$ to the domain $\frac{\pi}{4} < \mathrm{Arg} w < \pi$.