All curves are positively oriented unless otherwise noted.

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1. Find the integral
$$\oint_C \frac{3z^3+2}{(z-1)(z^2+9)}\,dz$$
where C is the circle $|z|=4$. Same for the circle $|z-2|=2$.

2. If C is any circle which does not pass through the points $0, \pm 1$ how many different values can assume the integral
$$\oint_C \frac{1}{z+1}+\frac{10}{z}+\frac{100}{z-1}\,dz?$$

3. (a) If the function $f:\mathbb{C}\setminus\{0\}\to\mathbb{C}$ is continuous and even and C is the circle $|z|=1$ show that
$$\oint_C f(z)\,dz = 0.$$
(b) If, in addition, the function is analytic in $\mathbb{C}\setminus\{0\}$ show the same if C is any simple closed curve going once around 0.
(c) Find a contiuous even function and a curve C as in (b) such that
$$\oint_C f(z)\,dz \neq 0.$$

4. Assume f is analytic at z_0 and has a zero there of order m. Show that the function $g(z)=f'(z)/f(z)$ has a simple pole at z_0 with residue m.