

All curves are positively oriented unless otherwise noted.

1. Find all the Laurent series (in all annuli defined by the singularities of the functions) around the point z_0 given.

$$(a) f(z) = \frac{1}{1-z}, z_0 = 0, \quad (b) f(z) = \frac{1}{1-z}, z_0 = 2, \quad (c) f(z) = e^{1/z}, z_0 = 0, \quad (d) f(z) = \frac{z}{(z-1)(z-3)}, z_0 = 1.$$

2. Suppose you have the Laurent expansion

$$f(z) = \sum_{n=-\infty}^{\infty} a_n z^n, \quad (r < |z| < R).$$

Find the Laurent expansion of the function

$$g(z) = (az^2 + bz + c)f(z).$$